Right Triangles in Algebra

Chapter 11

Where You're Going

In this chapter, you will learn how to

- Find the square roots of numbers.
- Find the missing measures of right triangles.
- Use the Distance and Midpoint Formulas.
- Solve a problem by writing a proportion.

Real-World Snapshots Applying what you learn, on pages 626–627 you will apply ratios to solve problems about proportions in rectangles.

LESSONS

Key Vocabulary

- angle of depression (p. 616)
- angle of elevation (p. 614)
- cosine (p. 608)
- distance (p. 592)
- hypotenuse (p. 584)
- irrational number (p. 581)
- legs (p. 584)
- midpoint (p. 594)
- perfect square (p. 580)
- sine (p. 608)
- square root (p. 580)
- tangent (p. 608)
- trigonometric ratio (p. 608)
- trigonometry (p. 608)

11-1 Square Roots and Irrational Numbers

OBJECTIVE 1

Finding Square Roots

What You'll Learn

- **OBJECTIVE 1** To find square roots of numbers
- **OBJECTIVE 2** To classify real numbers

. . . And Why

To use square roots in real-world situations, such as finding the distance to the horizon

✔ Check Skills You'll Need

Write the numbers in each list without using exponents.

1. $1^2, 2^2, 3^2, \dots, 12^2$

2. $10^2, 20^2, 30^2, \dots, 120^2$

For help, go to Lesson 4-2.

New Vocabulary

- perfect square
- square root
- irrational number

Reading Math

The symbol $\sqrt{100}$ is the positive square root of 100, so you may read $-\sqrt{100}$ as the negative square root of 100.

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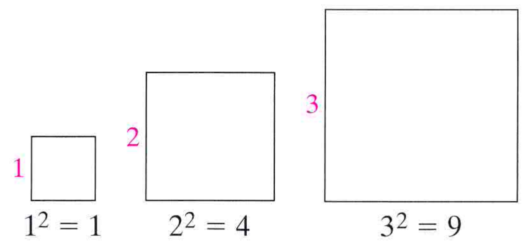
Consider the three squares shown below.

Each square has sides with integer length. The area of a square is the *square* of the length of a side. The square of an integer is a **perfect square.**



The inverse of squaring a number is finding a **square root.** The square-root radical, $\sqrt{\ }$, indicates the nonnegative square root of a number. In this book, you may assume that an expression under a square-root radical is greater than or equal to zero.

1 EXAMPLE Simplifying Square Roots

Simplify each square root.

a. $\sqrt{64}$

$\sqrt{64} = 8$

b. $-\sqrt{121}$

$-\sqrt{121} = -11$

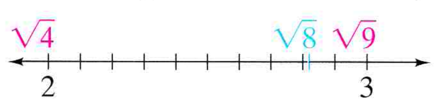
✔ Check Understanding Example 1

1. Simplify each square root.

a. $\sqrt{100}$ **b.** $-\sqrt{100}$ **c.** $\sqrt{16}$ **d.** $-\sqrt{16}$

The first thirteen perfect squares are
0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, and 144.
Memorizing these will help you solve problems efficiently.

For an integer that is not a perfect square, you can estimate a square root. For example, 8 is between the perfect squares 4 and 9.



$\sqrt{8}$ is between $\sqrt{4}$ and $\sqrt{9}$.

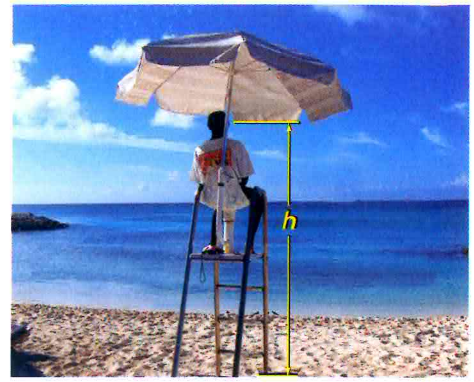
Since 8 is closer to 9 than to 4, $\sqrt{8}$ is closer to 3 than to 2. So, $\sqrt{8} \approx 3$.

2 EXAMPLE Real-World Problem Solving

Lifeguarding **You can use the formula $d = \sqrt{1.5h}$ to estimate the distance d, in miles, to a horizon line when your eyes are h feet above the ground. Estimate the distance to the horizon seen by a lifeguard whose eyes are 10 feet above the ground.**

$d = \sqrt{1.5h}$	Use the formula.
$d = \sqrt{15}$	Replace h with 10 and multiply.
$\sqrt{9} < \sqrt{15} < \sqrt{16}$	Find perfect squares close to 15.
$\sqrt{16} = 4$	Find the square root of the closest perfect square.

The lifeguard can see about 4 miles to the horizon.



Real-World Connection

This lifeguard can see nearly 1 mile farther than he can standing on the beach.

Check Understanding Example 2

2. Estimate to the nearest integer.

a. $\sqrt{27}$ **b.** $-\sqrt{72}$ **c.** $\sqrt{50}$ **d.** $-\sqrt{22}$

OBJECTIVE

2 Classifying Real Numbers

You can express a rational number as the ratio of two integers $\frac{a}{b}$, with $b \neq 0$. In decimal form, a rational number either terminates or repeats. An **irrational number** has decimal form that neither terminates nor repeats, and it cannot be written as the ratio of two integers. Together, rationals and irrationals form the real numbers.

If an integer is not a perfect square, its square root is irrational.

Need Help?

To review rational numbers, see Lesson 4-6.

3 EXAMPLE Identifying Irrational Numbers

Identify each number as rational or irrational. Explain.

a. $\sqrt{18}$	irrational, because 18 is not a perfect square
b. $\sqrt{121}$	rational, because 121 is a perfect square
c. **432.8**	rational, because it is a terminating decimal
d. **0.1212 . . .**	rational, because it is a repeating decimal
e. **0.120120012 . . .**	irrational; it neither terminates nor repeats
f. π	irrational; it cannot be represented as $\frac{a}{b}$, where a and b are integers

Check Understanding Example 3

3. Identify each number as rational or irrational. Explain.

a. $\sqrt{2}$ **b.** $-\sqrt{81}$ **c.** 0.53 **d.** $\sqrt{42}$

EXERCISES

For more exercises, see *Extra Practice.*

Practice and Problem Solving

A Practice by Example

Example 1 (page 580)

Simplify each square root.

1. $\sqrt{4}$ 2. $-\sqrt{36}$ 3. $\sqrt{1}$ 4. $\sqrt{25}$

5. $-\sqrt{49}$ 6. $\sqrt{81}$ 7. $-\sqrt{9}$ 8. $-\sqrt{169}$

Example 2 (page 581)

Estimate to the nearest integer.

9. $\sqrt{10}$ 10. $\sqrt{17}$ 11. $-\sqrt{39}$ 12. $-\sqrt{55}$

13. **Viewing Distance** The observation windows at the top of the Washington Monument in Washington, D.C., are 500 ft high. Using the formula $d = \sqrt{1.5h}$ of Example 2, estimate the distance you can see to the horizon from the observation windows.

Example 3 (page 581)

Identify each number as rational or irrational. Explain.

14. 4.1010010001 . . . 15. $\sqrt{87}$

16. $-\sqrt{16}$ 17. $-0.\overline{3}$

18. $\sqrt{5}$ 19. 2,222,222

20. $\sqrt{144}$ 21. 0.31311 . . .

B Apply Your Skills

Simplify each square root.

22. $\sqrt{196}$ 23. $\sqrt{\frac{4}{9}}$ 24. $\sqrt{\frac{25}{49}}$ 25. $\sqrt{\frac{36}{64}}$

Estimate to the nearest integer.

26. $\sqrt{7}$ 27. $\sqrt{2}$ 28. $\sqrt{40}$ 29. $-\sqrt{80}$

30. $\sqrt{58}$ 31. $-\sqrt{98}$ 32. $\sqrt{14}$ 33. $\sqrt{105}$

Identify each number as rational or irrational. Explain.

34. $\sqrt{0}$ 35. 1.001001001 . . . 36. 2.3010010001 . . .

37. **Reasoning** What do you get when you square $\sqrt{x}$?

38. **Writing in Math** A classmate was absent for today's lesson. Explain to him or her how to estimate $\sqrt{30}$.

39. a. **Patterns** You can create irrational numbers. For example, the number 1.010010001 . . . shows a pattern, yet it is irrational. What pattern do you see?
 b. **Open-Ended** Name three irrational numbers between 9 and 10.

Algebra **Find two integers that make each equation true.**

40. $a^2 = 9$ **41.** $b^2 = 25$ **42.** $y^2 = 100$ **43.** $m^2 = \frac{100}{25}$

44. **Geometry** Find the length of a side of a square with area 81 cm^2.

C Challenge

45. **Geometry** The area of a circle is 12 $in.^2$. Estimate its radius to the nearest inch.

If a number is the product of three identical factors, each factor is the *cube root* of the number. Since $2^3 = 8$, 2 is the cube root of 8. In Exercises 46–49, find the number *n* that makes each equation true.

46. $n^3 = -8$ **47.** $n^3 = 27$ **48.** $n^3 = -27$ **49.** $n^3 = 343$

On your graphing calculator, press MATH 4 to show the cube-root radical $\sqrt[3]{\ }$. Use it to find each cube root.

50. $\sqrt[3]{-8}$ **51.** $\sqrt[3]{125}$ **52.** $\sqrt[3]{-1{,}331}$ **53.** $\sqrt[3]{15{,}625}$

Test Prep

Multiple Choice

Joe is hiking in national park wilderness. His radio transmits signals as far as the horizon. Use the formula $d = \sqrt{1.5h}$, where d is the line-of-sight distance to the horizon in miles and h is height in feet, to answer the following questions.

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54. Joe has the transmitter at a height of 6 ft. About how far is it from the transmitter to the horizon?
A. 3 mi **B.** 5 mi **C.** 7 mi **D.** 9 mi

55. Joe sets his receiver on the ground. What is the farthest away that a transmitter 4 ft above the ground can be in order to reach Joe?
F. 0 mi **G.** 1.2 mi **H.** 2.4 mi **I.** 6 mi

Short Response

56. Joe wants to talk with his base camp that he knows is 6 mi away. **(a)** Explain how Joe could find the height at which he should have the transmitter. **(b)** Find that height.

Mixed Review

Lesson 10-9 **Find the volume of each figure in cubic centimeters.**

57. sphere with $r = 0.03$ m **58.** cone with $r = 4$ cm, $h = 10$ cm

Lesson 6-5

59. Shannon scored 17 correct on a 25-item test. The passing grade was 65%. Did Shannon pass? Explain.

Lesson 4-1 **List the positive factors of each number.**

60. 18 **61.** 22 **62.** 33 **63.** 45 **64.** 50 **65.** 90

11-2 The Pythagorean Theorem

OBJECTIVE 1

Using the Pythagorean Theorem

What You'll Learn

- **OBJECTIVE 1** To use the Pythagorean Theorem
- **OBJECTIVE 2** To identify right triangles

. . . And Why

To use the Pythagorean Theorem in real-world situations, such as carpentry

Check Skills You'll Need

Simplify.

1. $4^2 + 6^2$ 2. $5^2 + 8^2$
3. $7^2 + 9^2$ 4. $9^2 + 3^2$

For help, go to Lesson 4-2.

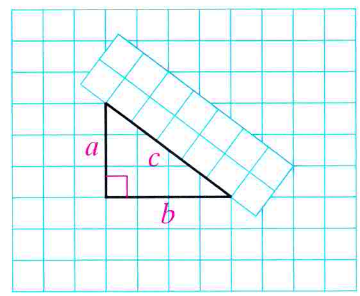
New Vocabulary

- legs
- hypotenuse

Investigation

Exploring Right Triangles

1. On graph paper, create right triangles with legs a and b. Measure the length of the third side c with another piece of graph paper.

   Copy and complete the table below.

a	b	c	a^2	b^2	c^2
3	4	■	9	16	■
5	12	■	25	144	■
9	12	■	81	144	■

2. Based on your table, use >, <, or = to complete the following statement.

$$a^2 + b^2 \;■\; c^2$$

Need Help?

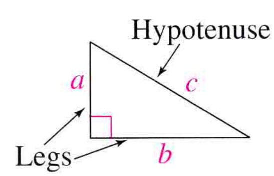
A right triangle is a triangle with a 90° angle.

In a right triangle, the two shortest sides are **legs.** The longest side, which is opposite the right angle, is the **hypotenuse.** The Pythagorean Theorem shows how the legs and hypotenuse of a right triangle are related.

Key Concepts — Pythagorean Theorem

In any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

$$a^2 + b^2 = c^2$$



You will prove the Pythagorean Theorem in a future math class. For now, you will use the theorem to find the length of a leg or the length of a hypotenuse.

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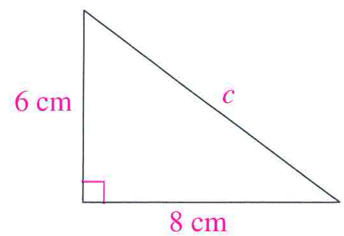
1 EXAMPLE Using the Pythagorean Theorem

Find c, the length of the hypotenuse, in the triangle at the right.



$c^2 = a^2 + b^2$	**Use the Pythagorean Theorem.**
$c^2 = 6^2 + 8^2$	**Replace a with 6 and b with 8.**
$c^2 = 100$	**Simplify.**
$c = \sqrt{100} = 10$	**Find the positive square root of each side.**

The length of the hypotenuse is 10 cm.

Check Understanding Example 1

1. The lengths of two sides of a right triangle are given. Find the length of the third side.

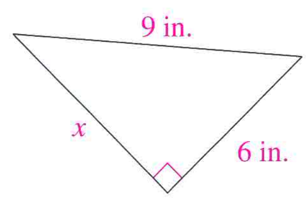
   a. legs: 3 ft and 4 ft

   b. leg: 12 m; hypotenuse: 15 m

You can use a calculator or a table of square roots to find approximate values for square roots.

2 EXAMPLE Finding an Approximate Length

Find the value of x in the triangle at the right. Round to the nearest tenth.



$a^2 + b^2 = c^2$	**Use the Pythagorean Theorem.**
$6^2 + x^2 = 9^2$	**Replace a with 6, b with x, and c with 9.**
$36 + x^2 = 81$	**Simplify.**
$x^2 = 45$	**Subtract 36 from each side.**
$x = \sqrt{45}$	**Find the positive square root of each side.**

Then use one of the two methods below to approximate $\sqrt{45}$.

Method 1 Use a calculator.
A calculator value for $\sqrt{45}$ is 6.7082039.

$x \approx 6.7$ **Round to the nearest tenth.**

Method 2 Use a table of square roots.
Use the table on page 778. Find 45 in the N column. Then find the corresponding value in the $\sqrt{N}$ column. It is 6.708.

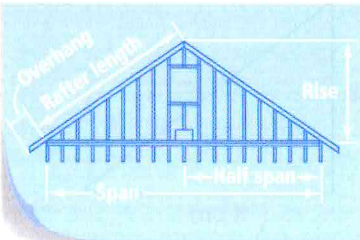
$x \approx 6.7$ **Round to the nearest tenth.**

The value of x is about 6.7 in.

Check Understanding Example 2

2. In a right triangle, the length of the hypotenuse is 15 m and the length of a leg is 8 m. What is the length of the other leg, to the nearest tenth of a meter?

3 EXAMPLE Real-World Problem Solving



Carpentry **The carpentry terms *span, rise,* and *rafter length* are illustrated in the diagram at the left. A carpenter wants to make a roof that has a span of 24 ft and a rise of 8.5 ft. What should the rafter length be?**

$c^2 = a^2 + b^2$	**Use the Pythagorean Theorem.**
$c^2 = 12^2 + 8.5^2$	**Use half the span, 12 ft. Replace *a* with 12 and *b* with 8.5.**
$c^2 = 144 + 72.25$	**Square 12 and 8.5.**
$c^2 = 216.25$	**Add.**
$c = \sqrt{216.25}$	**Find the positive square root.**
$c \approx 14.7$	**Round to the nearest tenth.**

The rafter length should be about 14.7 ft.

Check Understanding Example 3

3. **Carpentry** What is the rise of a roof if the span is 22 feet and the rafter length is 14 feet? Round to the nearest tenth of a foot.

OBJECTIVE 2 Identifying Right Triangles

The *Converse of the Pythagorean Theorem* allows you to substitute the lengths of the sides of a triangle into the equation $a^2 + b^2 = c^2$ to check whether a triangle is a right triangle. If the equation is true, the triangle is a right triangle.

4 EXAMPLE Finding a Right Triangle

Is a triangle with sides 12 m, 15 m, and 20 m a right triangle?

$a^2 + b^2 = c^2$	**Write the equation for the Pythagorean Theorem.**
$12^2 + 15^2 \stackrel{?}{=} 20^2$	**Replace *a* and *b* with the shorter lengths and *c* with the longest length.**
$144 + 225 \stackrel{?}{=} 400$	**Simplify.**
$369 \neq 400$	

The triangle is not a right triangle.

Check Understanding Example 4

4. Can you form a right triangle with the three lengths given? Explain.

a. 7 in., 8 in., $\sqrt{113}$ in. **b.** 5 mm, 6 mm, 10 mm

EXERCISES

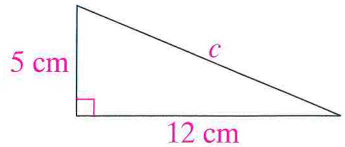
For more exercises, see *Extra Practice.*

Practice and Problem Solving

A Practice by Example

Examples 1 and 2 (page 585)

In each right triangle, find each missing length to the nearest tenth.

1.


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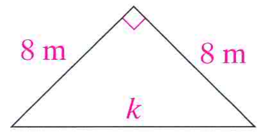

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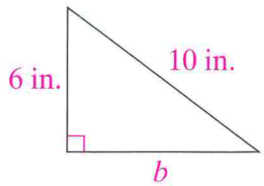

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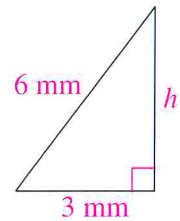


The lengths of two sides of a right triangle are given. Find the length of the third side. Round to the nearest tenth where necessary.

5. legs: 12 in. and 16 in.

6. legs: 21 ft and 28 ft

7. leg: 48 ft; hypotenuse: 50 ft

8. leg: 33 ft; hypotenuse: 55 ft

Example 3 (page 586)

Use the Pythagorean Theorem to solve each problem.

9. Carpentry Use the diagram in Example 3 on page 586. A carpenter wants to make a roof that has a rise of 5 ft and a rafter length of 16 ft. What is the half span? The span?

10. House Painting A painter places an 11-ft ladder against a house. The base of the ladder is 3 ft from the house. How high on the house does the ladder reach?

11. Hiking Darla hikes due north for 6 km. She then turns due east and hikes 3 km. What is the direct distance between her starting point and stopping point, rounded to the nearest tenth of a kilometer?

Example 4 (page 586)

Can you form a right triangle with the three lengths given? Explain.

12. 4 m, 6 m, 7 m

13. 4 mi, 5 mi, 6 mi

14. 7 in., 24 in., 25 in.

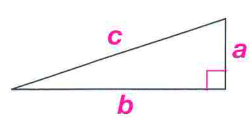
15. 6, 7, $\sqrt{85}$

16. 8 in., 10 in., 12 in.

17. 5 cm, 12 cm, 13 cm

B Apply Your Skills

Use the triangle at the right. Find the missing length to the nearest tenth of a unit.



18. $a = 2$ in., $b = 4$ in., $c = ▪$

19. $a = 1.4$ m, $b = 2.8$ m, $c = ▪$

20. $a = 3$ ft, $c = 5$ ft, $b = ▪$

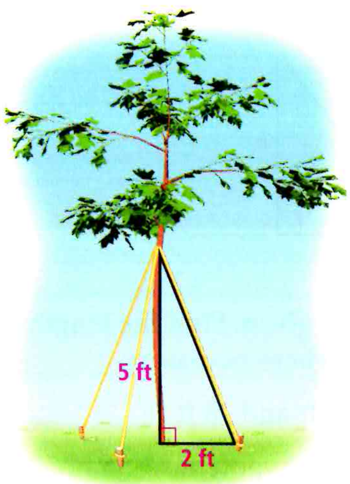
21. $b = 2.7$ km, $c = 3.4$ km, $a = ▪$

22. Reasoning Is a triangle with side lengths of $\sqrt{12}$ cm, $\sqrt{7}$ cm, and $\sqrt{5}$ cm a right triangle? Explain.

Any three positive integers that make $a^2 + b^2 = c^2$ true form a *Pythagorean triple*. Does each group of three integers below form a Pythagorean triple? Show your work.

23. 3, 4, 5 **24.** 7, 24, 25 **25.** 10, 24, 25 **26.** 5, 12, 13

27. For each group in Exercises 23–26 that forms a Pythagorean triple, multiply the integers by 2. Do the three new numbers form a Pythagorean triple? Show your work.



28. Landscaping Jim works for a landscaping company. He must plant and stake a tree. The stakes are 2 ft from the base of the tree. They are connected to wires that attach to the trunk at a height of 5 ft. If there is 6 in. of extra length at both ends of each wire, how long must each wire be, to the nearest tenth of a foot?

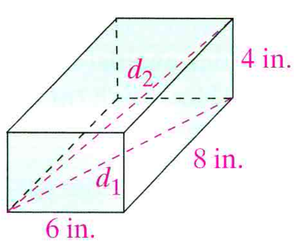
Can you form a right triangle having the three given lengths? Explain.

29. $\sqrt{5}$ yd, $\sqrt{3}$ yd, $\sqrt{2}$ yd **30.** 1 m, 0.54 m, 0.56 m

31. Quilting The diagonals for a quilting frame must be the same length to ensure the frame is rectangular. What should the lengths of the diagonals be for a quilting frame 86 in. by 100 in.? Draw a sketch and then solve.

32. Writing in Math Can you form a right triangle having side lengths of $3p$ ft, $4p$ ft, and $5p$ ft? Explain.

C Challenge



33. Geometry In the rectangular prism at the left, d_1 is the diagonal of the base of the prism, and d_2 is the *diagonal of the prism.*

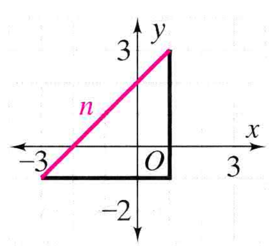
a. Find d_1.

b. The triangle formed by d_1, d_2, and the side that is 4 in. is a right triangle. Use your answer to part (a) to find d_2.

c. Find the diagonal of a rectangular prism with dimensions 9 in., 12 in., and 5 in.

Find the value of *n* in each diagram. Give your answer as a square root.

34.



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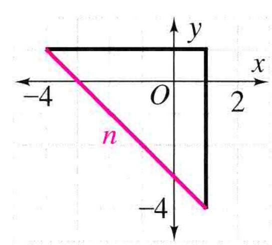
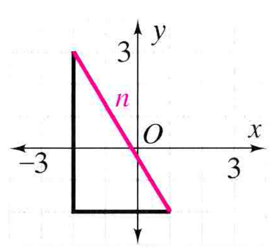
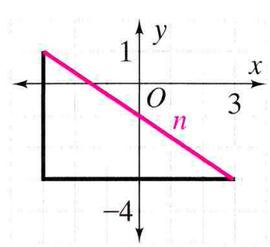


36.



37.



Test Prep

Multiple Choice

For Exercises 38 and 39, the lengths of two sides of a right triangle are given. What is the length of the third side?

38. legs: 36 m and 48 m
A. 5 m **B.** 12 m **C.** 50 m **D.** 60 m

39. leg: 6 m; hypotenuse: $\sqrt{85}$ m
F. 7 m **G.** 8 m **H.** 9 m **I.** 10 m

Extended Response

40. **a.** Can segments with lengths 3 ft, 4 ft, and 5 ft form a right triangle? Explain.
b. Ancient builders are said to have used ropes with 12 equally-spaced knots to make sure that square corners were indeed right angles. How could you use such a rope to determine whether a corner in your room forms a right angle?
c. Explain your answer in part (b).

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Mixed Review

Lesson 11-1 **Identify each number as rational or irrational.**

41. $\sqrt{36}$ **42.** $0.\overline{6}$ **43.** $-\sqrt{12}$ **44.** -33.3 **45.** $0.\overline{654}$

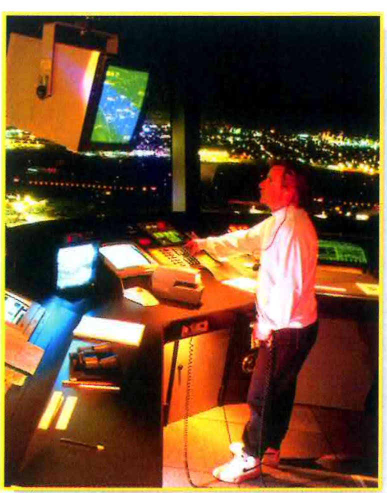
Lesson 5-9 **Simplify each expression.**

46. $(bc)^5$ **47.** $(2x^2)^4$ **48.** $(-3b)^3$ **49.** $(a^5b^2)^4$ **50.** $\left(\frac{3m}{5}\right)^2$

Lesson 4-9 **51.** **Geography** Greenland is the world's largest island and has an area of 2,175,600 km^2. Express this area in scientific notation.

Math at Work

Air-Traffic Controller



When we think of airline safety, many of us think of pilots. But there is also a network of people, the air-traffic controllers, who work hard to ensure the safe operation of aircraft. Using radar and visual observation, they closely monitor the location of each plane. They coordinate the movement of air traffic to make certain that aircraft stay a safe distance apart. They also coordinate landings and takeoffs to keep delays at a minimum.

In their jobs, air-traffic controllers use angle measurements in some of the same ways you do when you solve problems in algebra and geometry.

Take It to the NET For more information about air-traffic controllers, go to **www.PHSchool.com.**
Web Code: adb-2031

Extension

The Pythagorean Theorem and Circles

For Use With Lesson 11-2

Follow the steps below to discover a characteristic of circle chords and their perpendicular bisectors.

Step 1 With a compass, construct a large circle. Label the center O.

Step 2 Draw a chord $\overline{AB}$ that is not a diameter.

Step 3 Construct the perpendicular bisector of the chord with a compass and straightedge or by folding the circle so that A lies on B.

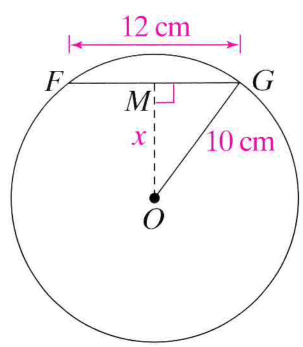
Step 4 Label the point where the perpendicular bisector intersects the chord as point D.

1. Write a conjecture about the perpendicular bisector of a chord and the center of the circle.
2. Classify $\triangle AOD$ by its angles.

The distance from the center of a circle to a chord is the length of the perpendicular segment with endpoints at the center and on the chord. You can use the radius of a circle and the length of a chord to find the distance from the center of a circle to the chord.

1 EXAMPLE

Circle O has a radius of 10 cm. Chord FG is 12 cm long. $\overline{OM}$ is the perpendicular bisector of $\overline{FG}$. How far is $\overline{FG}$ from O?



$\triangle OMG$ is a right triangle with legs $\overline{OM}$ and $\overline{MG}$, and hypotenuse $\overline{OG}$.

OM = distance to chord = x

OG = radius = 10 cm

$MG = \frac{1}{2}FG = \frac{1}{2}(12) = 6$

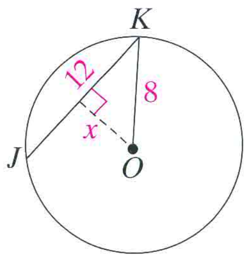
Use the Pythagorean Theorem to find x.

$$OG^2 = OM^2 + MG^2$$
$$10^2 = x^2 + 6^2$$
$$100 = x^2 + 36$$
$$100 - 36 = x^2 + 36 - 36$$
$$64 = x^2$$
$$8 = x$$

The distance from the center to the chord is 8 cm.

Find x, the distance from the center O of each circle to chord $\overline{JK}$. Round to the nearest tenth.

3.



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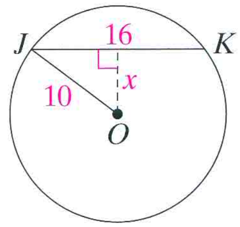

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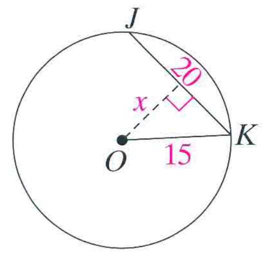
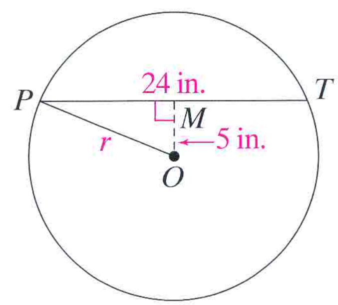


For a given circle, you can also find the length of a chord or the length of the radius if you know two other lengths.

2 EXAMPLE

Chord PT is 24 in. long and 5 in. from the center O of the circle. Find the length of the radius.

Use the Pythagorean Theorem to find the radius r.

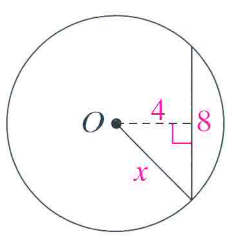


$$PM = \tfrac{1}{2}(PT) = 12$$
$$r^2 = 12^2 + 5^2$$
$$r^2 = 144 + 25$$
$$r^2 = 169$$
$$r = 13$$

The radius is 13 in.

Find x. If your answer is not an integer, round to the nearest tenth.

6.



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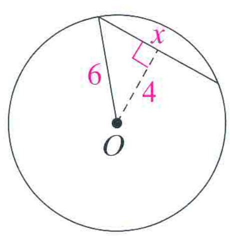

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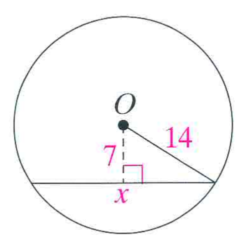

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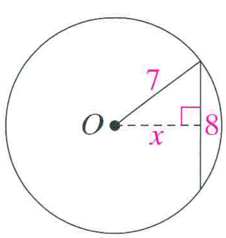

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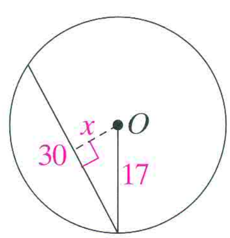

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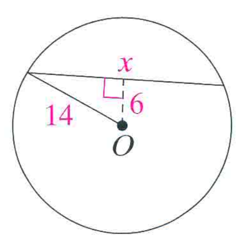


11-3 Distance and Midpoint Formulas

What You'll Learn

OBJECTIVE 1 To find the distance between two points using the Distance Formula

OBJECTIVE 2 To find the midpoint of a segment using the Midpoint Formula

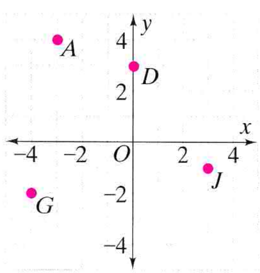
. . . And Why

To find the perimeters of figures on the coordinate plane

Check Skills You'll Need

Write the coordinates of each point.

1. A **2.** D **3.** G **4.** J



For help, go to Lesson 1-10.

New Vocabulary

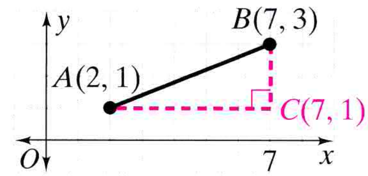
- distance
- midpoint

Interactive lesson includes instant self-check, tutorials, and activities.

OBJECTIVE 1

Finding Distance

In the graph at the right, you can locate point $C(7, 1)$ to form a right triangle with points $A(2, 1)$ and $B(7, 3)$. Using the Pythagorean Theorem, you can find AB.



$(AB)^2 = (AC)^2 + (BC)^2$

$(AB)^2 = (7 - 2)^2 + (3 - 1)^2$ **AC equals the difference in x values. BC equals the difference in y values.**

$(AB)^2 = 5^2 + 2^2$ **Simplify.**

$AB = \sqrt{25 + 4} = \sqrt{29} \approx 5.4$ **Find the square root.**

You can use the Pythagorean Theorem to find the length of a segment on a coordinate plane, or you can use the *Distance Formula.* The Distance Formula is based on the Pythagorean Theorem.

Key Concepts Distance Formula

You can find the **distance** d between any two points (x_1, y_1) and (x_2, y_2):

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

1 EXAMPLE Using the Distance Formula

Find the distance between $A(6, 3)$ and $B(1, 9)$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ **Use the Distance Formula.**

$d = \sqrt{(1 - 6)^2 + (9 - 3)^2}$ **Replace (x_2, y_2) with (1, 9) and (x_1, y_1) with (6, 3).**

$d = \sqrt{(-5)^2 + 6^2}$ **Simplify.**

$d = \sqrt{61}$ **Find the exact distance.**

$d \approx 7.8$ **Round to the nearest tenth.**

The distance between A and B is about 7.8 units.

Check Understanding Example 1

1. Find the distance between the two points in each pair. Round to the nearest tenth.

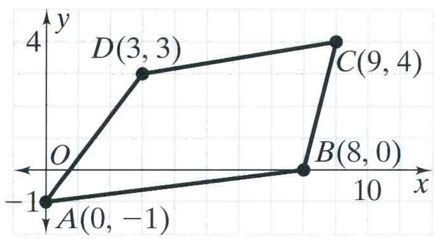
   a. (3, 8), (2, 4) b. (10, −3), (1, 0)

> **Reading Math**
> The Distance Formula indicates that you subtract (twice), square (twice), and add before you find the square root.

You can also use the Distance Formula to solve geometry problems. Wait until the last step to round your answer.

2 EXAMPLE Finding Perimeter

Find the perimeter of *ABCD*.



Use the Distance Formula to find the side lengths.

$AB = \sqrt{(8-0)^2 + (0-(-1))^2}$ **Replace (x_2, y_2) with (8, 0) and (x_1, y_1) with (0, −1).**

$= \sqrt{64 + 1} = \sqrt{65}$ **Simplify.**

$BC = \sqrt{(9-8)^2 + (4-0)^2}$ **Replace (x_2, y_2) with (9, 4) and (x_1, y_1) with (8, 0).**

$= \sqrt{1 + 16} = \sqrt{17}$ **Simplify.**

$CD = \sqrt{(3-9)^2 + (3-4)^2}$ **Replace (x_2, y_2) with (3, 3) and (x_1, y_1) with (9, 4).**

$= \sqrt{36 + 1} = \sqrt{37}$ **Simplify.**

$DA = \sqrt{(0-3)^2 + ((-1)-3)^2}$ **Replace (x_2, y_2) with (0, −1) and (x_1, y_1) with (3, 3).**

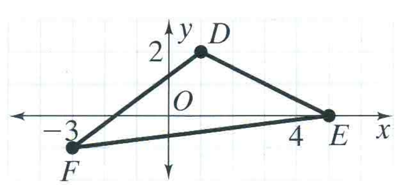
$= \sqrt{9 + 16} = \sqrt{25} = 5$ **Simplify.**

perimeter $= \sqrt{65} + \sqrt{17} + \sqrt{37} + 5 \approx 23.268126$

The perimeter is about 23.3 units.

Check Understanding Example 2

2. Find the perimeter of $\triangle DEF$ at the right. Round to the nearest tenth.



OBJECTIVE

2 Finding the Midpoint

The **midpoint** of a segment $\overline{AB}$ is the point M on $\overline{AB}$ halfway between the endpoints A and B where $AM = MB$.

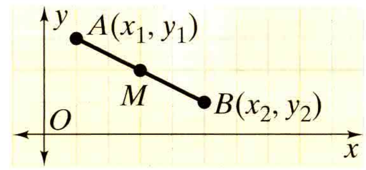
Reading Math

Each coordinate of a midpoint is the mean of the corresponding coordinates of the endpoints.

Key Concepts Midpoint Formula

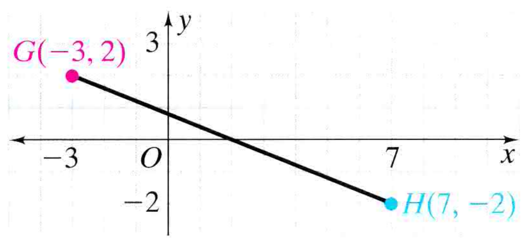
You can find the midpoint of a line segment with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$:

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



3 EXAMPLE Finding the Midpoint of a Segment

Find the midpoint of $\overline{GH}$.



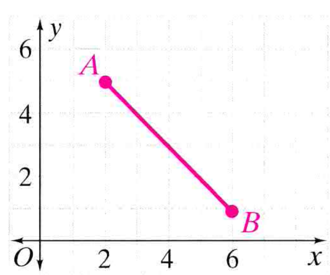
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	**Use the Midpoint Formula.**
$= \left(\frac{\ + 7}{2}, \frac{2 + (-2)}{2}\right)$	**Replace (x_1, y_1) with $(-3, 2)$ and (x_2, y_2) with $(7, -2)$.**
$= \left(\frac{4}{2}, \frac{0}{2}\right)$	**Simplify the numerators.**
$= (2, 0)$	**Write the fractions in simplest form.**

The coordinates of the midpoint of $\overline{GH}$ are $(2, 0)$.

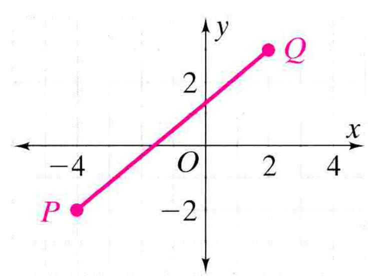
Check Understanding Example 3

3. Find the midpoint of each segment.

a.



b.



EXERCISES

For more exercises, see *Extra Practice.*

Practice and Problem Solving

A Practice by Example

Example 1 (page 592)

Find the distance between the two points of each pair. Round to the nearest tenth.

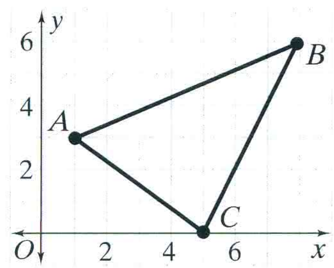
1. $(1, 5), (5, 2)$ **2.** $(6, 0), (-6, 5)$ **3.** $(-5, 10), (11, -7)$

4. $(-6, 12), (-3, -7)$ **5.** $(8, -1), (-5, 11)$ **6.** $(12, 3), (-12, 4)$

Example 2 (page 593)

Geometry **Find the perimeter of each figure.**

7.



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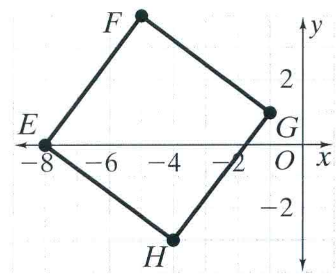
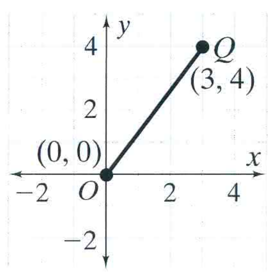


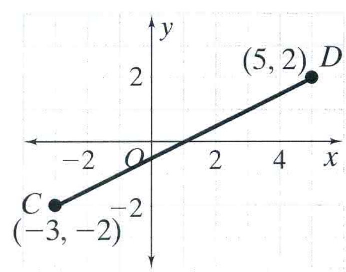
Example 3 (page 594)

Find the midpoint of each segment.

9.



10.



B Apply Your Skills

For Exercises 11 and 12, find both the length and the midpoint of the segment joining the two given points. Round to the nearest tenth.

11. $S(9, 12)$ and $U(-9, -12)$ **12.** $K(23, 4)$ and $W(-2, 16)$

13. **Error Analysis** A student's calculation of the midpoint of the segment with endpoints $A(-4, 2)$ and $B(6, 6)$ is shown at the right. What mistake did the student make?

$$\left(\frac{(6-(-4))}{2}, \frac{(6-2)}{2}\right)$$
$$=\left(\frac{10}{2}, \frac{4}{2}\right)$$
$$=(5, 2)$$

14. **Reasoning** The midpoint of $\overline{AB}$ is $(3, 5)$. The coordinates of A are $(-6, 1)$. What are the coordinates of B?

When you use the indicated formula, does it matter which point you choose as (x_1, y_1)? Explain.

15. the Midpoint Formula **16.** the Distance Formula

Writing in Math

For help with writing a solution for Exercise 17, see page 597.

17. A segment has endpoints $A(-3, 5)$ and $B(2, 1)$.
  a. Find the midpoint M of the segment.
  b. Use the Distance Formula to verify that $AM = MB$.

Challenge

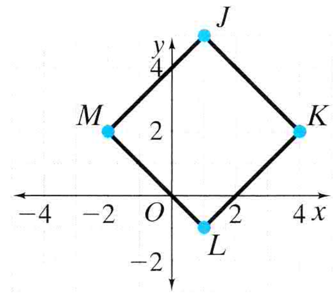
18. **Geometry** The three vertices of a triangle have coordinates $P(-3, 1)$, $Q(2, -5)$, and $R(4, 6)$. Determine whether the triangle is scalene, isosceles, or equilateral. Show your work.

19. **Writing in Math** Explain how using the Midpoint Formula involves finding averages.

Test Prep

Gridded Response

Use the diagram for Exercises 20 and 21.



20. What is the number of square units in the square?

21. What is the perimeter of the square, to the nearest tenth?

22. What is the length, to the nearest tenth, of the segment whose endpoints are $A(3, 7)$ and $B(8, 21)$?

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Web Code: ada-1103

Mixed Review

Lesson 11-2 **Can you form a right triangle with the lengths given?**

23. 8 m, 15 m, 17 m
24. 5 in., 8 in., 5 in.
25. 20 yd, 12 yd, 16 yd

Lesson 10-4

26. **Geometry** Draw a net to represent a rectangular prism that is 4 in. long, 3 in. wide, and 2 in. high. Label dimensions on the net.

Lesson 6-2 **Solve each proportion.**

27. $\frac{3}{8} = \frac{a}{24}$
28. $\frac{11}{c} = \frac{66}{72}$
29. $\frac{5}{6} = \frac{n}{15}$
30. $\frac{b}{1.9} = \frac{7}{9.5}$

Checkpoint Quiz 1 — Lessons 11-1 through 11-3

iTEXT Instant self-check quiz online and on CD-ROM

Estimate to the nearest integer.

1. $-\sqrt{3}$
2. $\sqrt{14}$
3. $\sqrt{27}$
4. $\sqrt{90}$
5. $-\sqrt{45}$
6. $\sqrt{105}$

The lengths of two legs of a right triangle are given. Find the length of the hypotenuse. Round to the nearest tenth.

7. 6 ft, 8 ft
8. 8 m, 14 m
9. 7 yd, 24 yd
10. 5 cm, 5 cm

Find the length of $\overline{AB}$ and the midpoint of $\overline{AB}$. Round the length of $\overline{AB}$ to the nearest tenth.

11. $A(0, -2)$ and $B(-6, -9)$
12. $A(8, 11)$ and $B(-5, 2)$

13. **Open-Ended** Name three irrational numbers between 10 and 20.

Writing in Math

Writing to Justify

For Use With Page 595, Exercise 17

One way to justify your solution of a problem is to give a reason for each step you take, keeping in mind the persons who will read your work. Acceptable reasons include properties and procedures that you and your audience have agreed upon.

EXAMPLE

A segment has endpoints $A(-3, 5)$ and $B(2, 1)$.

a. Find the midpoint M of the segment.

b. Use the Distance Formula to verify that $AM = MB$.

a. First find the midpoint of $\overline{AB}$.

Steps	Reasons
$\left(\frac{-3+2}{2}, \frac{5+1}{2}\right)$	**Use the Midpoint Formula. Replace (x_1, y_1) with $(-3, 5)$ and (x_2, y_2) with $(2, 1)$.**
$\left(-\frac{1}{2}, 3\right)$ or $(-0.5, 3)$	**Simplify.**

b. Next verify that this is the midpoint M by showing that $AM = MB$.

Steps	Reasons
$AM = \sqrt{(-0.5 - (-3))^2 + (3 - 5)^2}$	**Find *AM*. Use the Distance Formula.**
$AM = \sqrt{10.25}$	**Simplify.**
$MB = \sqrt{(2 - (-0.5))^2 + (1 - 3)^2}$	**Find *MB*. Use the Distance Formula.**
$MB = \sqrt{10.25}$	**Simplify.**

Since AM and MB are both $\sqrt{10.25}$, $AM = MB$ and $(-0.5, 3)$ is the midpoint of $\overline{AB}$.

EXERCISES

Solve each problem. Justify your steps to verify your answer.

1. Find the perimeter of the triangle with vertices located at $(5, 9)$, $(7, 4)$, and $(-3, 7)$.

2. The vertices of a triangle are located at $(-1, 5)$, $(2, 5)$, and $(2, 1)$. Show that this is a right triangle. (*Hint:* Use the Distance Formula and the Converse of the Pythagorean Theorem.)

11-4 Problem Solving
Write a Proportion

What You'll Learn

OBJECTIVE 1 To write a proportion from similar triangles

. . . And Why

To solve problems of unknown distance

Check Skills You'll Need

Solve each proportion.

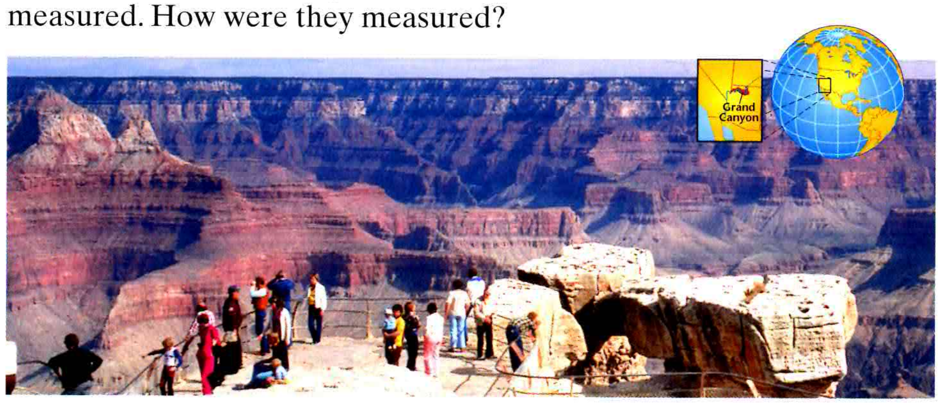
1. $\frac{1}{3} = \frac{a}{12}$
2. $\frac{h}{5} = \frac{20}{25}$
3. $\frac{1}{4} = \frac{8}{x}$
4. $\frac{2}{7} = \frac{c}{35}$

For help, go to Lesson 6-2.

OBJECTIVE

1 Write a Proportion

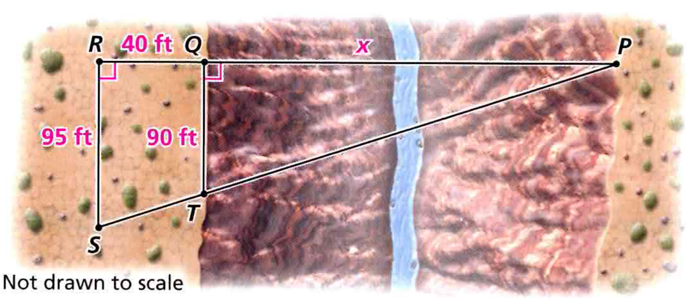
Math Strategies in Action You can't measure distance across the Grand Canyon with a tape measure. Yet, distances across it have been measured. How were they measured?



Surveyors sometimes find such distances indirectly using similar triangles and proportions. You learned about these in Lessons 6-2 and 6-3. Now let's see how you can use similar right triangles and proportions to find measurements indirectly.

1 EXAMPLE Real-World Problem Solving

Surveying To find the distance from Q to P across a canyon, a surveyor picks points R and S such that $\overline{RS}$ is perpendicular to $\overline{RP}$. He locates point T on $\overline{SP}$ such that $\overline{QT}$ is perpendicular to $\overline{RP}$. The two triangles, $\triangle PRS$ and $\triangle PQT$, are similar. He then measures $\overline{RS}$, $\overline{RQ}$, and $\overline{QT}$. What is the distance QP across the canyon?



Not drawn to scale

Test-Taking Tip

If problems involving distance do not include diagrams, draw your own to help you solve the problem.

Read and Understand

1. What information is given?
2. What are you asked to find?

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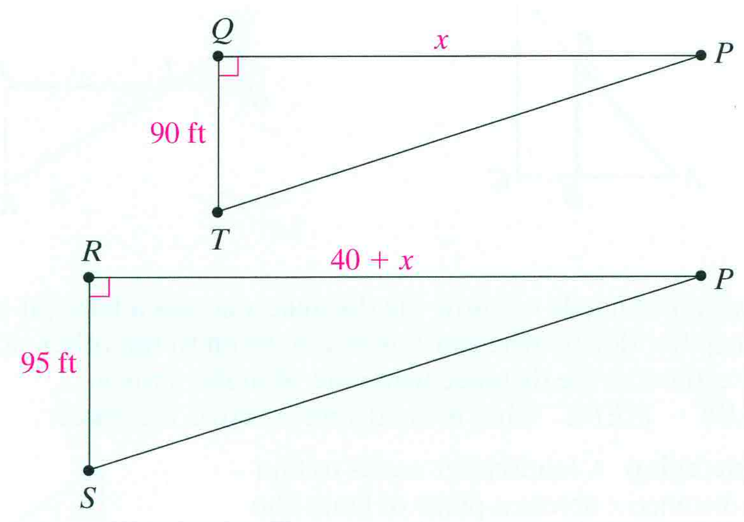
Plan and Solve

Since $\triangle PQT \sim \triangle PRS$ and you know three lengths, writing and solving a proportion is a good strategy to use. It is helpful to draw the triangles as separate figures.



Write a proportion using the legs of the similar right triangles.

$\frac{x}{40 + x} = \frac{90}{95}$	**Write a proportion.**
$95x = 90(40 + x)$	**Write cross products.**
$95x = 3{,}600 + 90x$	**Use the Distributive Property.**
$5x = 3{,}600$	**Subtract 90*x* from each side.**
$x = 720$	**Divide each side by 5.**

The distance QP across the canyon is 720 ft.

Look Back and Check

Solving problems that involve indirect measurement often makes use of figures that *overlap*. Use the diagram on page 598 to answer the following questions.

Check Understanding

3. Which segments overlap?

4. A common error students make is to use part of a side in a proportion. For example, some students might think $\frac{40}{95}$ is equal to $\frac{x}{90}$. How does drawing the triangles as separate figures help you avoid this error?

EXERCISES

For more exercises, see *Extra Practice.*

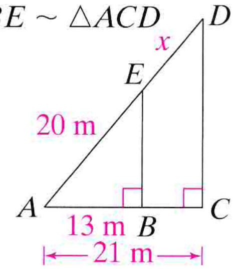
Practice and Problem Solving

A Practice by Example

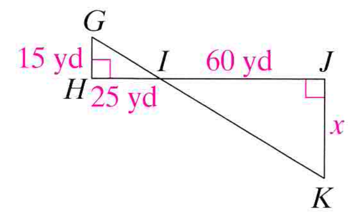
Example 1 (page 598)

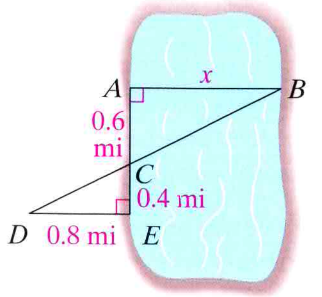
In Exercises 1–5, write a proportion and find the value of each x.

1. $\triangle ABE \sim \triangle ACD$



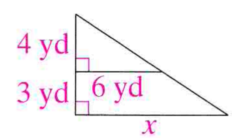
2. $\triangle GHI \sim \triangle KJI$





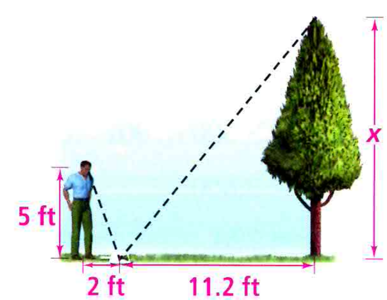
3. A swimmer needs to know the distance x across a lake (at left) to help her decide whether it is safe to swim to the other side. She estimates the distance using the triangles shown. $\triangle ABC \sim \triangle EDC$. What is the distance across the lake?

4. Landscaping A landscaper needs to find the distance x across a piece of land. She estimates the distance using the similar triangles at the right. What is the distance?



B Apply Your Skills

5. Indirect Measurement To estimate the height of a tree, Milton positions a mirror on the ground so he can see the top of the tree reflected in it. His height, his distance from the mirror, and his line of sight to the mirror determine a triangle. The tree's height, its distance from the mirror, and the distance from the top of the tree to the mirror form a similar triangle. Use the measurements shown to determine the height of the tree.



Solve using any strategy.

6. There are 30 students in a math class. Twelve belong to the computer club, and eight belong to the photography club. Three belong to both clubs. How many belong to neither club?

7. Jake spent $\frac{3}{8}$ of his money on a book and $\frac{1}{2}$ of what was left on a magazine. He now has \$6.25. How much money did he start with?

8. Hai takes 12 minutes to walk to school. He wants to get there 15 minutes early to meet with his lab partner. What time should he leave his house if school starts at 8:10 A.M.?

9. Number Sense Christa thought of a number. She added 4, multiplied the sum by -5, and subtracted 12. She then doubled the result and got -34. What number did Christa start with?

Strategies

- Account for All Possibilities
- Draw a Diagram
- Look for a Pattern
- Make a Model
- Make a Table
- Simplify the Problem
- Simulate the Problem
- Solve by Graphing
- Try, Test, Revise
- Use Multiple Strategies
- Work Backward
- Write an Equation
- Write a Proportion

C Challenge

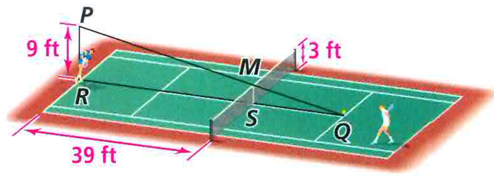
Real-World Connection

The Eiffel Tower was named for its designer, Gustave Eiffel, who also designed the Statue of Liberty framework.

10. The height of the Eiffel Tower is 984 ft. A souvenir model of the tower is 6 in. tall. At 5 P.M. in Paris, the shadow of the souvenir model is 8 in. long. The Eiffel Tower and its shadow determine two legs of a right triangle that are similar to the two legs of a right triangle determined by the souvenir model and its shadow. About how long is the shadow of the Eiffel Tower?

11. Architecture Madison Square Garden in New York City is built in the shape of a circle. Its diameter is 404 ft and it accommodates 20,234 spectators. To the nearest tenth of a square foot, how much area is there for each spectator?

12. **Algebra** You serve a tennis ball from one end of a tennis court, 39 ft from the net. You hit the ball at 9 feet above the ground. It travels in a straight path down the middle of the court, and just clears the top of the 3-ft net. This is illustrated in the diagram. $\triangle PQR \sim \triangle MQS$. How far from the net does the ball land?



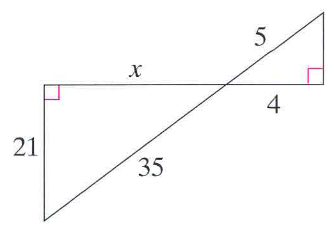
Test Prep

Multiple Choice

13. In the diagram at the right, what is x?

A. 43.75 **B.** 28

C. 26.25 **D.** 17



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14. What is the value of x in this proportion, $\frac{65}{50} = \frac{x + 36}{x}$?

F. 15 **G.** 60 **H.** 120 **I.** 156

Short Response

15. Suppose a friend says to you "x is to $x + 1.5$ as 64 is to 72."

a. Write a proportion to solve for x.

b. Solve your proportion.

Mixed Review

Lesson 11-3 **Find the midpoint of each segment with given endpoints.**

16. $A(2, 3)$ and $B(4, 7)$ **17.** $L(-1, 2)$ and $M(2, 6)$

Lessons 9-3 **Sketch each figure.**

18. isosceles right triangle **19.** scalene obtuse triangle

Lesson 7-4 **20. Fundraising** Keith collected twice as much money as Lucy for a walkathon. Together they collected $120. How much money did each person collect?

11-5 Special Right Triangles

What You'll Learn

OBJECTIVE 1 To use the relationships in 45°-45°-90° triangles

OBJECTIVE 2 To use the relationships in 30°-60°-90° triangles

. . . And Why

To find distances in real-world situations, such as in sports

Check Skills You'll Need

Find the missing side of each right triangle.

1. legs: 6 m and 8 m

2. leg: 9 m; hypotenuse: 15 m

3. legs: 27 m and 36 m

4. leg: 48 m; hypotenuse: 60 m

For help, go to Lesson 11-2.

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OBJECTIVE

1 Using 45°-45°-90° Triangles

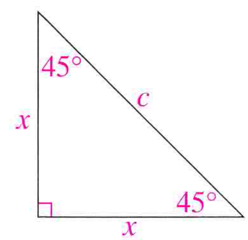
The Pythagorean Theorem requires that you understand square roots. The rule for Multiplying Square Roots will help you work with square roots more efficiently.

Key Concepts Multiplying Square Roots

For nonnegative numbers, the square root of a product equals the product of the square roots.

Arithmetic	Algebra
$\sqrt{9 \cdot 2} = \sqrt{9} \cdot \sqrt{2}$	If $a \geq 0$ and $b \geq 0$, then $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

The rule for Multiplying Square Roots is especially useful with an isosceles right triangle, which is also known by its angle measures as a 45°-45°-90° triangle. You can use the rule to relate the lengths of the sides and the hypotenuse in such a triangle.



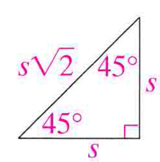
$c^2 = a^2 + b^2$	Use the Pythagorean Theorem.
$c^2 = x^2 + x^2$	Replace *a* and *b* with *x*.
$c^2 = 2x^2$	Simplify.
$c = \sqrt{2x^2}$	Find the square root.
$c = \sqrt{2} \cdot \sqrt{x^2}$	Use the rule for Multiplying Square Roots.
$c = \sqrt{2} \cdot x$, or $x\sqrt{2}$	Simplify.

This shows the following special relationship.

Key Concepts 45°-45°-90° Triangles

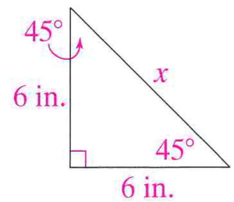
In a 45°-45°-90° triangle, the legs are congruent and the length of the hypotenuse is the length of a leg times $\sqrt{2}$.

$$\text{hypotenuse} = \text{leg} \cdot \sqrt{2}$$



1 EXAMPLE Finding Length of the Hypotenuse

Find the length of the hypotenuse in the triangle at the right.



hypotenuse $= \text{leg} \cdot \sqrt{2}$	**Use the 45°-45°-90° relationship.**
$x = 6 \cdot \sqrt{2}$	**The length of the leg is 6.**
≈ 8.5	**Use a calculator.**

The length of the hypotenuse is about 8.5 in.

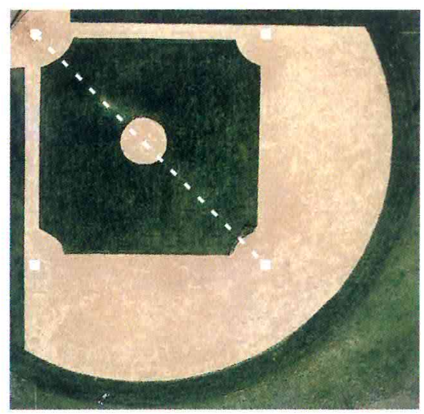
Check Understanding Example 1

1. The length of each leg of an isosceles right triangle is 4.2 cm. Find the length of the hypotenuse. Round to the nearest tenth.

You can use 45°-45°-90° triangles in real-world situations.

2 EXAMPLE Real-World Problem Solving

Baseball A baseball diamond is a square. The distance from any base to the next is 90 ft. How far is it from home plate to second base?



hypotenuse $= \text{leg} \cdot \sqrt{2}$	**Use the 45°-45°-90° relationship.**
$= 90 \cdot \sqrt{2}$	**The length of the leg is 90.**
≈ 127.28	**Use a calculator.**

The distance from home plate to second base is about 127 ft.

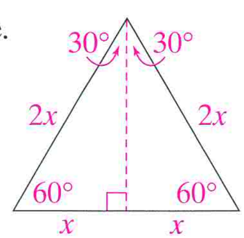
Check Understanding Example 2

2. Gymnasts use mats that are 12 m by 12 m for floor exercises. A gymnast does cartwheels across the diagonal of a mat. What is the length of the diagonal to the nearest meter?

OBJECTIVE

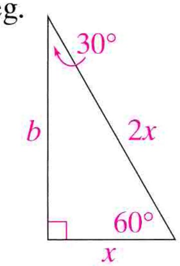
2 Using 30°-60°-90° Triangles

Another special right triangle is the 30°-60°-90° triangle. You can form two congruent 30°-60°-90° triangles by bisecting an angle of an equilateral triangle. This is shown in the diagram.



In the diagram, the length of the hypotenuse of each 30°-60°-90° triangle is twice the length of the shorter leg. You can use the Pythagorean Theorem to find the length of the longer leg.

For the figure at the right, find the length of the longer leg.

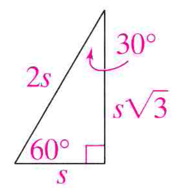


$$(2x)^2 = x^2 + b^2 \quad \textbf{Use the Pythagorean Theorem.}$$
$$4x^2 = x^2 + b^2 \quad \textbf{Simplify.}$$
$$3x^2 = b^2 \quad \textbf{Subtract } x^2 \textbf{ from each side.}$$
$$\sqrt{3x^2} = b \quad \textbf{Find the square root.}$$
$$\sqrt{3} \cdot \sqrt{x^2} = b \quad \textbf{Rule for Multiplying Square Roots}$$
$$b = \sqrt{3} \cdot x, \text{ or } x\sqrt{3} \quad \textbf{Simplify.}$$

This shows the special relationship of the hypotenuse and the legs in a 30°-60°-90° triangle.

Key Concepts 30°-60°-90° Triangle

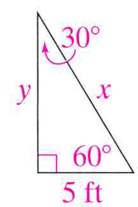
In a 30°-60°-90° triangle, the length of the hypotenuse is 2 times the length of the shorter leg. The length of the longer leg is the length of the shorter leg times $\sqrt{3}$.



$$\text{hypotenuse} = 2 \cdot \text{shorter leg}$$
$$\text{longer leg} = \text{shorter leg} \cdot \sqrt{3}$$

3 EXAMPLE Finding Lengths in a 30°-60°-90° Triangle

Find the missing lengths in the triangle.



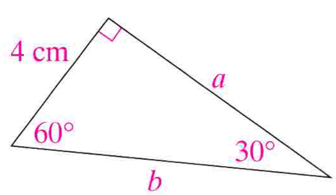
$$\text{hypotenuse} = 2 \cdot \text{shorter leg}$$
$$x = 2 \cdot 5 \quad \textbf{The length of the shorter leg is 5.}$$
$$x = 10 \quad \textbf{Simplify.}$$
$$\text{longer leg} = \text{shorter leg} \cdot \sqrt{3}$$
$$y = 5 \cdot \sqrt{3} \quad \textbf{The length of the shorter leg is 5.}$$
$$y \approx 8.7 \quad \textbf{Use a calculator.}$$

The length of the hypotenuse is 10 ft, and the length of the longer leg is about 8.7 ft.

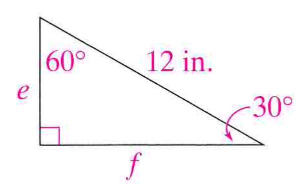
Check Understanding Example 3

3. Find the missing lengths in each 30°-60°-90° triangle.

a.



b.



EXERCISES

For more exercises, see *Extra Practice.*

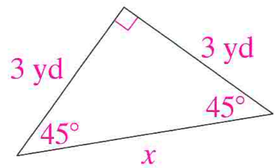
Practice and Problem Solving

A Practice by Example

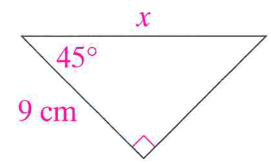
Example 1 (page 603)

The lengths of the legs of an isosceles right triangle are given. Find the length of each hypotenuse to the nearest tenth.

1.



2.



3. legs: 5.4 in.   **4.** legs: 17 ft   **5.** legs: 21 m

Example 2 (page 603)

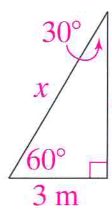
6. Ballet A ballet teacher wants to divide his square classroom in half diagonally with masking tape. How much tape will he need if the side length of the classroom is 40 ft?

7. Flooring A square piece of tile with sides 12 in. is cut along a diagonal. What is the length of the diagonal rounded to the nearest inch?

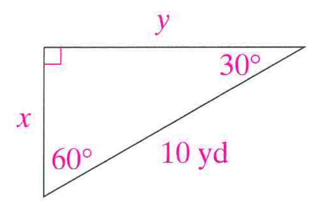
Example 3 (page 604)

Find the missing lengths. Round to the nearest tenth.

8.

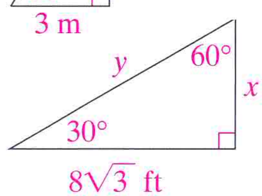


9.

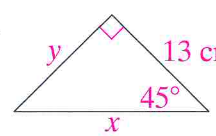


B Apply Your Skills

10.



11.



12. Error Analysis A student says that a right triangle with a hypotenuse of length $2\sqrt{2}$ in. has to be an isosceles right triangle. What mistake might the student have made?

13. Writing in Math Explain how to find the lengths of the longer leg and hypotenuse of a 30°-60°-90° triangle if the shorter leg is 10 ft.

14. Mrs. Fernandez wants to string a rope diagonally across her square classroom for her students to hang their completed art projects. If one side of the room measures 20 ft, what is the minimum length of rope she can use?

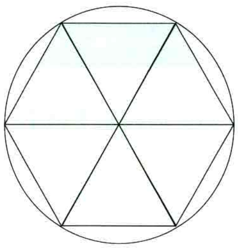
Simplify. Use the rule for Multiplying Square Roots.

15. $\sqrt{3} \cdot \sqrt{27}$ **16.** $\sqrt{50} \cdot \sqrt{2}$ **17.** $\sqrt{36} \cdot \sqrt{4}$

C Challenge

18. Reasoning The smaller angles of a 30°-60°-90° triangle are in the ratio 1 : 2. Are the shorter sides also in the ratio 1 : 2? Explain.

19. **Geometry** A polygon is inscribed in a circle if all of its vertices lie on the circle. To find the area of the hexagon inscribed in a circle with a diameter of 8 in., answer each of the following.



a. The segments shown form 6 congruent equilateral triangles. What is the length of each side of each triangle?
b. What is the height of one triangle?
c. What is the area of one triangle?
d. What is the area of the hexagon?

Writing in Math

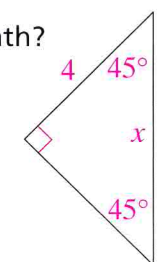
Sketch a 45°-45°-90° triangle. Use *x* for the length of one leg. Explain how to find the lengths of the other sides of the triangle.

20. **Geometry** You can inscribe a regular hexagon in a circle using a compass and straightedge.
a. Use your compass to construct a circle. Keep the compass at the same setting. Place the tip of the compass on the circle. Mark an arc on the circle. Place the tip of the compass where the arc intersects the circle and mark another arc. Continue around the circle until you have six arcs on the circle. Join consecutive arcs with segments.
b. Measure the diameter of the circle. Use this measure and Exercise 19 to find the area of your hexagon.

Test Prep

Multiple Choice

21. In the triangle at the right, what is *x*, to the nearest tenth?
A. 8.7 **B.** 8 **C.** 5.7 **D.** 2



22. In a 30°-60°-90° triangle, the length of the longer leg is 10. What is the length of the hypotenuse, to the nearest tenth?
F. 5.0 **G.** 5.8 **H.** 11.5 **I.** 17.3

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Short Response

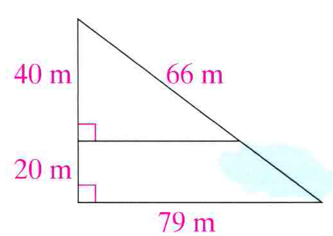
In Exercises 23–25, **(a)** tell whether a triangle with sides of the given lengths could be 45°-45°-90° or 30°-60°-90°. **(b)** Explain your answers.

23. 6, 8, 10 24. 5, 5, $5\sqrt{2}$ 25. 15, $7.5\sqrt{3}$, 7.5

Mixed Review

Lesson 11-4

26. **Surveying** A surveyor needs to find the distance across a lake. He estimates the distance using the similar triangles at the right. What is the distance?



Lesson 9-6

Find the circumference of each circle with the given radius or diameter.

27. radius = 4 in. 28. diameter = 6 m 29. radius = 2.5 ft

Lesson 8-1

Is each relation a function? Explain.

30. $\{(-1, 3), (0, 4), (1, 5)\}$ 31. $\{(2, 3), (3, 4), (4, 5), (5, 6)\}$

Extension

Square Roots of Expressions With Variables

For Use With Lesson 11-5

You can simplify square roots of expressions that contain variables. Assume that the value of each variable is not negative.

1 EXAMPLE

Write each square root without a radical sign.

a. $\sqrt{25x^2}$

$\sqrt{25x^2} = \sqrt{(5x)^2}$ Write $25x^2$ as the square of $5x$.

$= 5x$ Simplify.

b. $\sqrt{p^6}$

$\sqrt{p^6} = \sqrt{(p^3)^2}$ Use the rule for the Power of a Power.

$= p^3$ Simplify.

You can also simplify expressions that have nonsquare factors by using the rule for Multiplying Square Roots.

2 EXAMPLE

Simplify each square root.

a. $\sqrt{x^9}$

$\sqrt{x^9} = \sqrt{x^8 \cdot x}$ Use the rule for Multiplying Powers with the Same Base.

$= \sqrt{x^8} \cdot \sqrt{x}$ Use the rule for Multiplying Square Roots.

$= x^4\sqrt{x}$ Simplify.

b. $\sqrt{48x}$

$\sqrt{48x} = \sqrt{16 \cdot 3x}$ Find a perfect square factor.

$= \sqrt{16} \cdot \sqrt{3x}$ Use the rule for Multiplying Square Roots.

$= 4\sqrt{3x}$ Simplify.

EXERCISES

Write each square root without the radical sign.

1. $\sqrt{49y^2}$
2. $\sqrt{100m^{12}}$
3. $-\sqrt{25x^6}$
4. $\sqrt{a^2b^{10}}$
5. $-\sqrt{169w^{26}}$

Simplify each square root.

6. $\sqrt{a^{12}}$
7. $\sqrt{36x^4}$
8. $\sqrt{81b^8}$
9. $-\sqrt{64a^{16}}$
10. $-\sqrt{x^4y^{12}}$
11. $\sqrt{c^7}$
12. $\sqrt{x^{23}}$
13. $-\sqrt{20m}$
14. $\sqrt{27b^{11}}$
15. $-\sqrt{72a^{19}}$

11-6

Sine, Cosine, and Tangent Ratios

What You'll Learn

OBJECTIVE 1 To find trigonometric ratios in right triangles

OBJECTIVE 2 To use trigonometric ratios to solve problems

. . . And Why

To find lengths that cannot be measured directly

Check Skills You'll Need

Solve each problem.

1. A 6-ft man casts an 8-ft shadow while a nearby flagpole casts a 20-ft shadow. How tall is the flagpole?
2. When a 12-ft tall building casts a 22-ft shadow, how long is the shadow of a nearby 14-ft tree?

For help, go to Lesson 6-3.

New Vocabulary

- trigonometry
- trigonometric ratio
- sine
- cosine
- tangent

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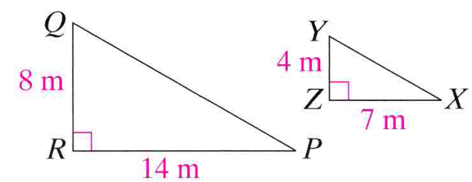
OBJECTIVE 1

Finding Ratios in Right Triangles

Investigation

Exploring Ratios in Similar Right Triangles

1. In the diagram at the right, $\triangle PQR \sim \triangle XYZ$. Find the length of the hypotenuse of each triangle.



2. In the triangles above, $\angle P$ is the smallest angle of $\triangle PQR$ and $\angle X$ is the smallest angle of $\triangle XYZ$. For each figure, write the following ratios in simplest form.

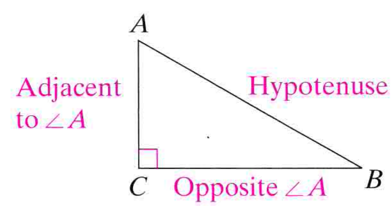
   a. $\dfrac{\text{length of leg opposite smallest angle}}{\text{length of hypotenuse}}$

   b. $\dfrac{\text{length of leg adjacent to smallest angle}}{\text{length of hypotenuse}}$

   c. $\dfrac{\text{length of leg opposite smallest angle}}{\text{length of leg adjacent to smallest angle}}$

3. What do you notice about the two ratios you wrote for each part of Question 2?

The word **trigonometry** means triangle measure. The ratio of the lengths of two sides of a right triangle is a **trigonometric ratio.** To write trigonometric ratios, you must identify sides that are opposite and adjacent to the acute angles of a triangle.



Trigonometric Ratios

sine $\angle A = \dfrac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \dfrac{CB}{AB}$

cosine $\angle A = \dfrac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \dfrac{AC}{AB}$

tangent $\angle A = \dfrac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \dfrac{CB}{AC}$

You can use these abbreviations when you find trigonometric ratios for a given acute $\angle N$.

$$\sin N = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos N = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan N = \frac{\text{opposite}}{\text{adjacent}}$$

Reading Math

The abbreviations for sine, cosine, and tangent are sin, cos, and tan, respectively.

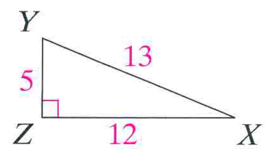
1 EXAMPLE Writing Trigonometric Ratios

For $\triangle XYZ$, find the sine, cosine, and tangent of $\angle X$.

$$\sin X = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{5}{13}$$

$$\cos X = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{12}{13}$$

$$\tan X = \frac{\text{opposite}}{\text{adjacent}} = \frac{5}{12}$$



Check Understanding Example 1

1. For $\triangle XYZ$ above, find the sine, cosine, and tangent of $\angle Y$.

Trigonometric ratios are usually expressed in decimal form as approximations. If you know the measure of an acute angle of a right triangle, you can use a calculator or a table of trigonometric ratios to find approximate values for the sine, cosine, and tangent of the angle.

2 EXAMPLE Using a Calculator

Find the trigonometric ratios of 42° using a calculator or the table on page 779. Round to four decimal places.

$\sin 42° \approx 0.6691$

$\cos 42° \approx 0.7431$

$\tan 42° \approx 0.9004$

Scientific calculator: Enter 42 and press the key labeled SIN, COS, or TAN.

Table: Find 42° in the first column. Look across to find the appropriate ratio.

Graphing Calculator Hint

If you use a graphing calculator, enter the trigonometric ratio name before you enter the angle measure. Be sure the calculator is in degree mode.

Check Understanding Example 2

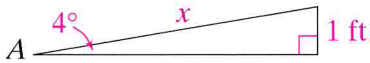
2. Find each value. Round to four decimal places.

a. $\sin 10°$ **b.** $\cos 75°$ **c.** $\tan 53°$ **d.** $\cos 22°$

OBJECTIVE 2 Using Trigonometric Ratios to Solve Problems

You can use trigonometric ratios to find measures in right triangles indirectly. The advantage to using trigonometric ratios is that you need only an acute angle measure and the length of one side to find the lengths of the other two sides.

3 EXAMPLE — Real-World Problem Solving



Not drawn to scale

Ramps **What is the length of the wheelchair ramp at the left?**

You know the angle and the side opposite the angle. You want to find x, the length of the hypotenuse.

$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$ **Use the sine ratio.**

$\sin 4^\circ = \frac{1}{x}$ **Substitute 4° for the angle and 1 for the side opposite.**

$x(\sin 4^\circ) = 1$ **Multiply each side by *x*.**

$x = \frac{1}{\sin 4^\circ}$ **Divide each side by sin 4°.**

$x \approx 14.3$ **Use a calculator.**

The ramp is about 14.3 ft long.

Check Understanding Example 3

3. How long is the longer leg under the ramp in Example 3?

More Than One Way

Ladders **Find the height, *x*, that the ladder reaches.**

Kevin's Method

Use the Pythagorean theorem.

$3^2 + x^2 = 8^2$

$9 + x^2 = 64$

$x^2 = 55$

$x = \sqrt{55} \approx 7.4$

The ladder reaches about 7.4 ft.

Tina's Method

Use a trigonometric ratio.

$\sin 68^\circ = \frac{x}{8}$

$8(\sin 68^\circ) = x$

$7.4 \approx x$

The ladder reaches about 7.4 ft.

Choose a Method

1. Which method do you prefer to use? Explain.

2. What information is needed for each method?

EXERCISES

For more exercises, see *Extra Practice.*

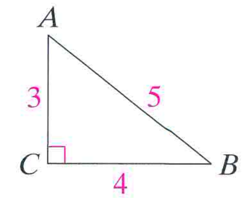
Practice and Problem Solving

A Practice by Example

For Exercises 1–3, find the length of the indicated side.

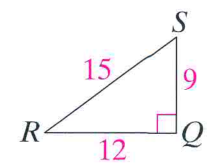
Example 1 (page 609)

1. the leg opposite $\angle A$
2. the leg adjacent to $\angle A$
3. the hypotenuse of $\triangle ABC$
4. Find the sine, cosine, and tangent of $\angle A$.



Use $\triangle QRS$ for Exercises 5 and 6.

5. Find the sine, cosine, and tangent of $\angle R$.
6. Find the sine, cosine, and tangent of $\angle S$.



Example 2 (page 609)

Find each value. Round to four decimal places.

7. tan 89° 8. sin 30° 9. cos 14° 10. sin 67°
11. tan 28° 12. cos 89° 13. tan 11° 14. sin 83°

Example 3 (page 610)

15. **Waterskiing** The angle that a waterskiing ramp forms with the surface of the water is 15°, and the ramp rises 5 ft. Approximately how long is the ramp?

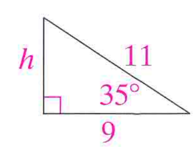
16. **Loading Ramps** The ramp on the back of a mover's truck is 12 ft long. If the angle of the ramp with the ground is 22°, about how high is the floor of the truck above the ground?

B Apply Your Skills

Use right triangles to find the ratios. Show your diagrams.

17. tan 45° 18. cos 60° 19. sin 30° 20. cos 30°

21. **Writing in Math** Tom says the missing length in the triangle at the right can be found using the tangent ratio. Jed says it can be found using the sine ratio. Who is correct? Explain.

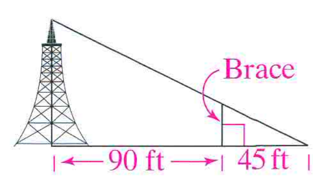


22. **Hot-Air Balloons** A hot-air balloon climbs continuously along a 30° angle to a height of 5,000 feet. To the nearest tenth of a foot, how far has the balloon traveled to reach 5,000 feet? Draw a sketch, and then solve.

C Challenge

23. **Reasoning** Find the sine of an angle and the cosine of its complement. Do this for several angles. Make a conjecture.

24. A wire from a radio tower is supported by a 23-ft brace.
    a. How tall is the radio tower?
    b. What is the measure of the angle formed by the wire and the ground?



Test Prep

Multiple Choice In Exercises 25 and 26, how high on a wall does a 12-ft ladder reach?

25. The base of the ladder is 3 ft from the wall.
A. 4.0 ft **B.** 10.0 ft **C.** 11.6 ft **D.** 12 ft

26. The ladder forms a 60° angle with the ground.
F 10.4 ft **G.** 8.0 ft **H.** 6.9 ft **I.** 6.0 ft

Reading Comprehension Read the passage below before doing Exercises 27 and 28.

The Tilting Tower

Building began on the bell tower at Pisa, Italy, in 1173. Shortly after that, the 55.9-m tower began to lean and has continued to lean even more over the centuries. In 1993, the tower had a tilt that was 5.5° from the vertical. In the spring of 1999, engineering experts began work at the base to correct some of the lean. They completed the work in 2001 after decreasing the lean by 0.5°. The engineers hope their work will stabilize the tower for the next 300 years.

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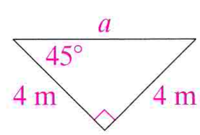
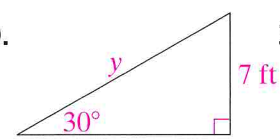
27. In 2001, what was the tilt of the tower in degrees?

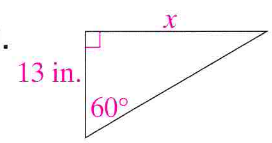
28. In 2001, about how many meters from the vertical was the top of the tower?

Mixed Review

Lesson 11-5 **Find the missing lengths.**

29.


30.


31.


Lesson 9-7 **32.** **Geometry** $\overleftrightarrow{GT}$ is the perpendicular bisector of $\overline{RA}$ at point E. Name two congruent segments.

Lesson 8-2 **Find the solutions of each equation for $x = -1, 0,$ and 3.**

33. $y = -3x + 2$ **34.** $x + y = 20$ **35.** $6x - 2y = 12$

Checkpoint Quiz 2 — Lessons 11-4 through 11-6

Instant self-check quiz online and on CD-ROM

Tell whether a triangle with sides of the given lengths is 45°-45°-90°, 30°-60°-90°, or neither.

1. $8, 8, 8\sqrt{2}$ **2.** $1, 2, \sqrt{3}$ **3.** $12\sqrt{3}, 12, 24$

Find each value. Round to four decimal places.

4. cos 61° **5.** tan 30° **6.** sin 32° **7.** sin 87°

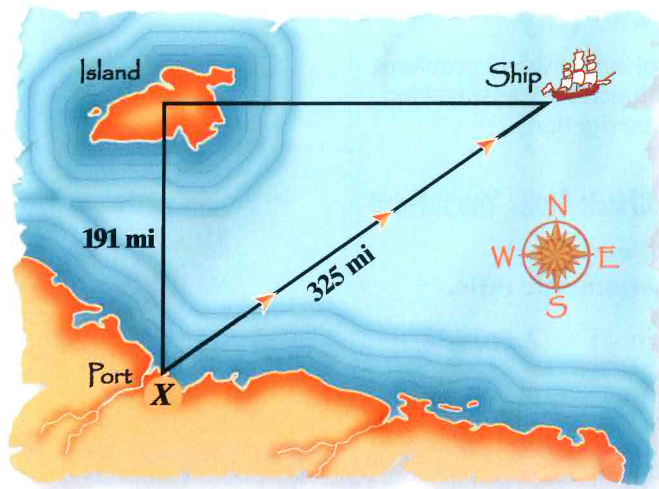
Finding Angle Measures

For Use With Lesson 11-6

You can use a calculator or a trigonometric-ratio table to find the degree measure of an acute angle of a right triangle if two sides of the triangle are known. If you are using a graphing calculator, be sure you are in degree mode.

EXAMPLE

You have a map charting a ship's course. The ship is traveling from the port along the course shown. What is the angle from due north of the ship's course?



The angle formed by due north, the port, and the ship is the angle at which the ship is traveling. This is angle X.

$\cos X = \frac{191}{325}$

To find $m\angle X$ with a calculator:

Press **TRIG cos^{-1}** 191 ÷ 325 **ENTER**.

$m\angle X \approx 54°$

To find $m\angle X$ using the table of trigonometric ratios on page 779:

$\cos X = \frac{191}{325} \approx 0.5877$ **Divide.**

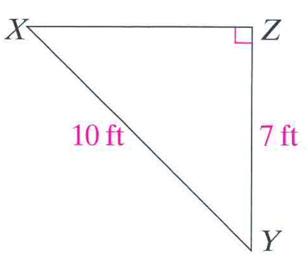
$m\angle X \approx 54°$ **In the cosine column, find the decimal closest to 0.5877, which is 0.5878. Read the angle measure across from 0.5878.**

The ship is traveling at a 54° angle from due north.

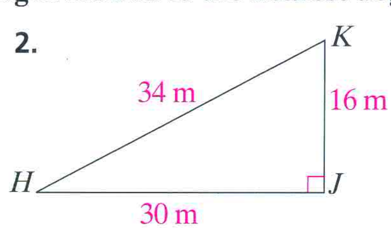
EXERCISES

Find the measure of each acute angle. Round to the nearest degree.

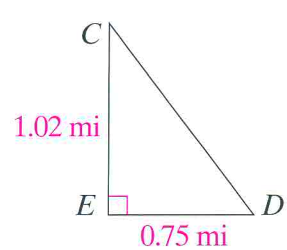
1.



2.



3.



In $\triangle PQR$, $\angle Q$ is a right angle. Find the measures of $\angle P$ and $\angle R$ to the nearest tenth.

4. $PQ = 3$, $PR = 5$

5. $RQ = 12$, $PR = 13$

6. $PQ = 20$, $RQ = 21$

11-7 Angles of Elevation and Depression

What You'll Learn

OBJECTIVE 1 To use trigonometry to find angles of elevation

OBJECTIVE 2 To use trigonometry to find angles of depression

. . . And Why

To solve real-world problems in subjects such as surveying and navigation

Check Skills You'll Need

Find each trigonometric ratio.

1. sin 45° **2.** cos 32°

3. tan 18° **4.** sin 68°

5. cos 88° **6.** tan 84°

For help, go to Lesson 11-6.

New Vocabulary

- angle of elevation
- angle of depression

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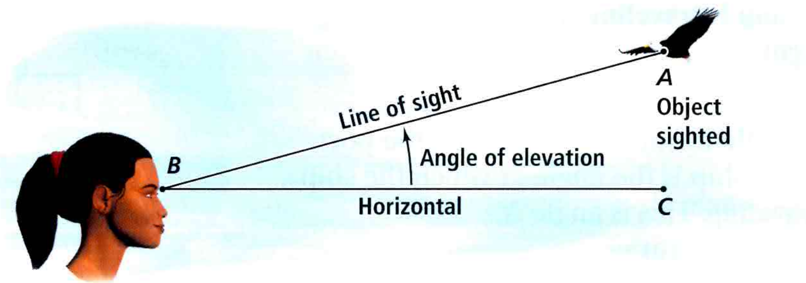
OBJECTIVE

1 Angles of Elevation

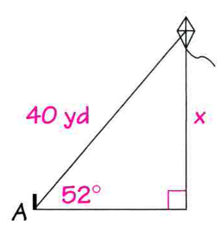
Civil engineers and navigators use the terms *angle of elevation* and *angle of depression* to describe the angles at which they observe things. An **angle of elevation** is formed by a horizontal line and a line of sight above it. It is used when you must look up at an object.



$\angle ABC$ is an angle of elevation.

1 EXAMPLE Real-World Problem Solving

Kite Flying **Marcus is flying a kite. He lets out 40 yd of string and anchors it to the ground. He determines that the angle of elevation of the kite is 52°. What is the height x of the kite from the ground?**



Draw a picture.

$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$ **Choose an appropriate trigonometric ratio.**

$\sin 52^\circ = \frac{x}{40}$ **Substitute 52° for the angle measure and 40 for the hypotenuse.**

$40(\sin 52^\circ) = x$ **Multiply each side by 40.**

$32 \approx x$ **Simplify.**

The kite is about 32 yd from the ground.

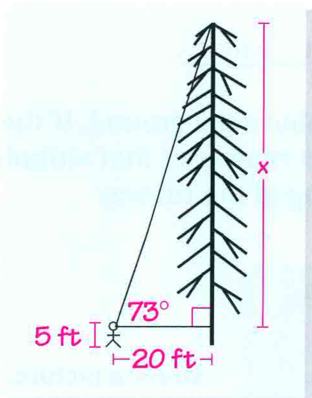
Check Understanding Example 1

1. The angle of elevation from a ship to the top of a lighthouse is 12°. The lighthouse is known to be 30 m tall. How far is the ship from the base of the lighthouse?

In real life, a person's line of sight is parallel to the ground at eye height. In some problems you must account for this.

2 EXAMPLE Real-World Problem Solving

Indirect Measurement **Felicia wants to determine the height of a tree. From her position 20 ft from the base of the tree, she sees the top of the tree at an angle of elevation of 73°. Felicia's eyes are 5 ft from the ground. How tall is the tree, to the nearest foot?**



Draw a picture.

$\tan = \frac{\text{opposite}}{\text{adjacent}}$	**Choose an appropriate trigonometric ratio.**
$\tan 73° = \frac{x}{20}$	**Substitute 73 for the angle measure and 20 for the adjacent side.**
$20(\tan 73°) = x$	**Multiply each side by 20.**
$65 \approx x$	**Use a calculator or a table.**
$65 + 5 = 70$	**Add 5 to account for the height of Felicia's eyes from the ground.**

The tree is about 70 ft tall.

Check Understanding Example 2

2. A rock climber looks at the top of a vertical rock wall at an angle of elevation of 74°. He is standing 4.2 m from the base of the wall and his eyes are 1.5 m from the ground. How high is the wall, to the nearest tenth of a meter?

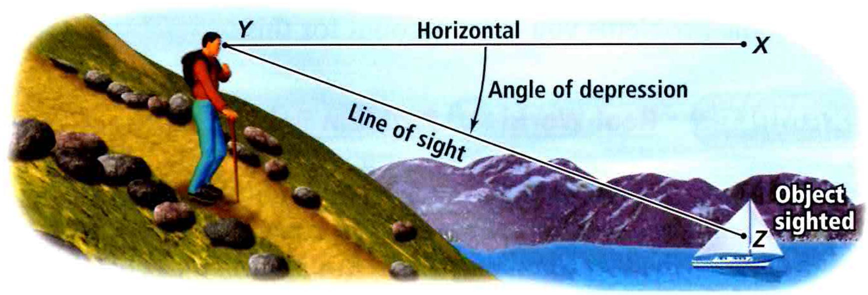
OBJECTIVE

2 Angles of Depression

Reading Math

With an angle of elevation, the object sighted is elevated, or above the horizontal line. With an angle of depression, the object is depressed, or below the horizontal line.

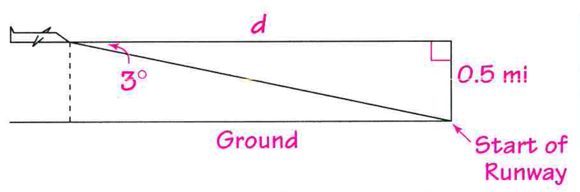
An **angle of depression** is formed by a horizontal line and a line of sight below it. It is used when you must look down at an object.



$\angle XYZ$ is an angle of depression.

3 EXAMPLE Real-World Problem Solving

Navigation **An airplane is flying 0.5 mi above the ground. If the pilot must begin a 3° descent to an airport runway at that altitude, how far is the airplane from the beginning of the runway (in ground distance)?**



Not drawn to scale

Draw a picture.

$\tan 3^\circ = \frac{0.5}{d}$ **Choose an appropriate trigonometric ratio.**

$d \tan 3^\circ = 0.5$ **Multiply each side by *d*.**

$\frac{d \tan 3^\circ}{\tan 3^\circ} = \frac{0.5}{\tan 3^\circ}$ **Divide each side by tan 3°.**

$d = \frac{0.5}{\tan 3^\circ}$ **Simplify.**

$d \approx 9.5$ **Use a calculator.**

The airplane is about 9.5 mi from the airport.

Check Understanding Example 3

3. A group of people in a hang-gliding class are standing on top of a cliff 70 m high. They spot a hang glider landing on the beach below them. The angle of depression from the top of the cliff to the hang glider is 72°. How far is the hang glider from the base of the cliff?

EXERCISES

For more exercises, see *Extra Practice.*

Practice and Problem Solving

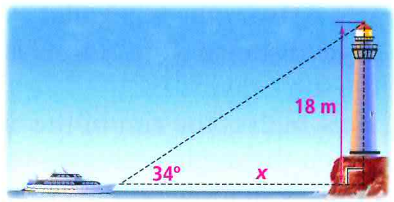
A Practice by Example

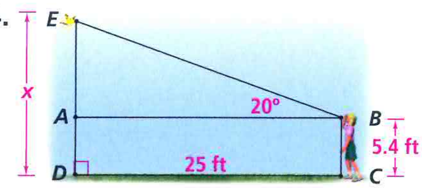
Examples 1 and 2 (pages 614 and 615)

1. In Example 1, Marcus's kite drops so that the angle of elevation is 48°. Find the height of the kite above the ground.

2. In Example 2, Felicia spots a nest in the tree at an angle of elevation of 65°. How high above the ground is the nest?

Find x to the nearest tenth.

3. 

4. 

Example 3 (page 616)

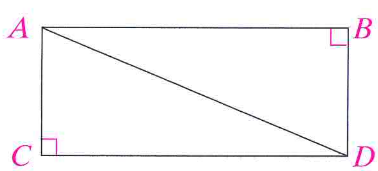
For Exercises 5 and 6, draw a sketch and solve.

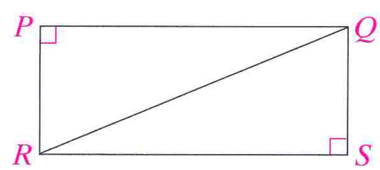
5. An airplane descends at an angle of 2°. Its altitude decreases by 2.5 miles. What is the ground distance covered by the airplane?

6. An airplane descends at an angle of 22.5° over a ground distance of 0.5 mi. By how many miles, to the nearest tenth, does its altitude decrease?

B Apply Your Skills

Name the angles of elevation and depression in each figure.

7. 

8. 

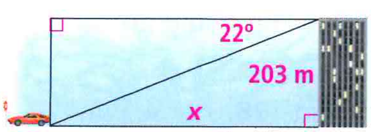
Reading Math

For help with reading and solving Exercise 9, see page 619.

9. **Bird Watching** A rare bird is spotted in a tree by a bird-watching group. The group is 9.5 yd from the base of the tree. The angle of elevation to the bird is 57°. How far is the bird from the group along the line of sight? Draw a sketch, and then solve.

10. **Navigation** The angle of elevation from a ship to the top of a lighthouse is 4°. If the top of the lighthouse is known to be 50 m above sea level, how far is the ship from the lighthouse?

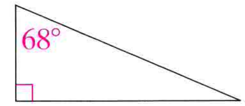
11. **Writing in Math** How do you decide which trigonometric ratio to use to solve a problem?



12. Find the distance x of the car from the office building at the left.

13. **Meteorology** A meteorologist measures the angle of elevation of a weather balloon as 53°. A radio signal from the balloon indicates that it is 1,620 m (line-of-sight) from the meteorologist's location. How high above ground is the weather balloon?

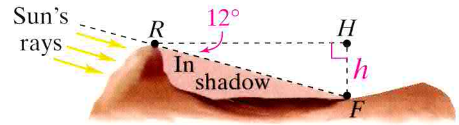
14. **Error Analysis** A student made the drawing at the right to solve an angle-of-depression problem. What mistake has the student made?



15. **Aeronautics** The pilot of a helicopter at an altitude of 6,000 ft sees a second helicopter at an angle of depression of 43°. The altitude of the second helicopter is 4,000 ft. What is the distance from the first helicopter to the second along the line of sight?

C Challenge

16. **Astronomy** The diagram suggests a method for finding depths of moon craters from Earth. Astronomers calculate that the distance from R to H is 3 km when the angle of depression of the sun's rays is 12°.



   a. How high is the rim of the crater from the floor of the crater?
   b. When the angle increases to 14°, how much shorter is the crater's shadow on the crater floor?

17. **Surveying** Surveyors find that a canyon is 4 km wide. From one canyon rim, the angle of depression to the base of the canyon wall below the other rim is 7°. The angle of depression to a river is 8°. How far is the river from the far canyon wall?

Test Prep

Multiple Choice

18. A ramp has an angle of elevation of 19° and a length of 35 ft. What is the height in inches of the ramp at its high end?
A. 11 in. **B.** 12 in. **C.** 137 in. **D.** 145 in.

19. A radio transmitting tower casts a shadow that extends 344 feet from the base of the tower. A line connecting the sun, the top of the tower, and the end of the shadow forms an angle of depression of 16°. Approximately how high is the tower?
F. 344 ft **G.** 99 ft **H.** 1,200 ft **I.** 1,248 ft

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Mixed Review

Lesson 11-6

Find each value. Round to four decimal places.

20. tan 29° **21.** sin 80° **22.** cos 34° **23.** sin 76° **24.** cos 45°

Lesson 10-3

Find the area of each circle. Give an exact area using π and an approximate area to the nearest tenth.

25. $r = 8$ in. **26.** $r = 1.9$ cm **27.** $r = 10$ mm **28.** $r = 4.5$ in.

Lesson 6-2

29. **Baking** A recipe that serves four people calls for $1\frac{1}{2}$ c of flour. How many cups of flour are needed to serve ten people?

Reading Math

Reading for Problem Solving

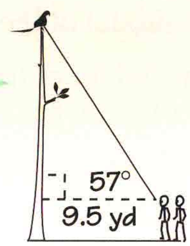
For Use With Page 617, Exercise 9

Read the problem. Then follow along with what Cheryl thinks as she solves it. Check your understanding by solving the exercise at the bottom of the page.

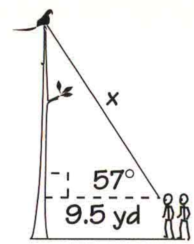
Bird Watching A rare bird is spotted in a tree by a bird-watching group. The group is 9.5 yd from the base of the tree. The angle of elevation to the bird is 57°. How far is the bird from the group along the line of sight? Draw a sketch, and then solve.

What Cheryl Thinks / What Cheryl Writes

I'll draw a sketch. In it, I'll show:
angle of elevation = 57°,
distance from tree = 9.5 yd,
right angle near base of tree.



I must find the line-of-sight distance to the bird. That's along the hypotenuse of the right triangle. I'll name it *x*.



I know an angle and the adjacent leg.
I want to find the hypotenuse.
I'll use the cosine ratio.

$$\cos 57° = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$= \frac{9.5}{x}$$

I can solve this equation.

$$x(\cos 57°) = 9.5$$
$$\frac{x(\cos 57°)}{\cos 57°} = \frac{9.5}{\cos 57°}$$
$$x = \frac{9.5}{\cos 57°}$$

I'll use a calculator and round.

$$x \approx 17.4$$

Now I can write the answer.

The bird is about 17.4 yd from the group along the line of sight.

EXERCISE

1. A hot-air balloon is 150 feet high. The angle of depression from the pilot to her assistant on the ground is 25°. What is the line-of-sight distance from the pilot to her assistant, to the nearest foot?

Test-Taking Strategies

Using a Variable

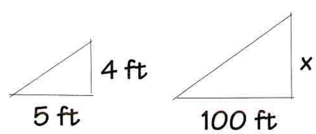
You can solve many problems by letting a variable be an unknown quantity you want to find. Use the variable in an equation. Then solve the equation to help solve the problem.

1 EXAMPLE

When José visited France he saw a large model of the Statue of Liberty. He wondered how tall the model was, so he measured its shadow. The shadow was 100 feet. He measured the height and shadow of a vertical pole. The pole measured 4 feet and its shadow was 5 feet. How tall was the model of the Statue of Liberty?

The unknown quantity is the height of the model. Sketch similar triangles (not to scale). Let x be the height. Then write a proportion using x.

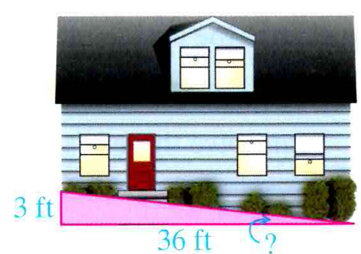
$\frac{5}{100} = \frac{4}{x}$	**Write a proportion.**
$5x = 100(4)$	**Write cross products.**
$5x = 400$	**Simplify.**
$x = 80$	**Divide each side by 5.**



The model was 80 ft tall.

2 EXAMPLE

City code requires that wheelchair ramps rise no more than 1 in. for every horizontal foot. To meet the code, you build a ramp the entire length of your house, 36 ft. What is the ramp's angle of elevation?



The unknown quantity is the measure of the angle that the ramp forms with the ground. Let x be that measure. You know the side opposite and the side adjacent the angle.

$\tan x = \frac{3}{36}$	**Use the tangent ratio.**
≈ 0.0833	**Write as a decimal.**
$x \approx 5$	**Use a calculator (see page 613).**

The angle of elevation of the ramp is about 5°.

EXERCISE

1. You are flying a kite at the end of 200 ft of string. The angle of elevation is 30°. You are holding the end of the string at eye level, 5 ft above the ground. How high is the kite from the ground? Draw a diagram. Show how to use a variable. Then find how high the kite is.

Chapter 11

Chapter Review

Vocabulary

angle of depression (p. 616)
angle of elevation (p. 614)
cosine (p. 608)
distance (p. 592)
hypotenuse (p. 584)
irrational number (p. 581)
legs (p. 584)
midpoint (p. 594)
perfect square (p. 580)
sine (p. 608)
square root (p. 580)
tangent (p. 608)
trigonometric ratio (p. 608)
trigonometry (p. 608)

Reading Math
Understanding Vocabulary

Match the vocabulary terms on the right with their descriptions on the left.

1. the two shortest sides of a right triangle
2. an angle formed by a horizontal line and a line of sight above it
3. the ratio of the lengths of two sides of a right triangle
4. the longest side of a right triangle, opposite the right angle
5. a number that cannot be expressed as a ratio of two integers
6. The inverse of squaring a number is finding this.

a. irrational number
b. square root
c. legs
d. trigonometric ratio
e. hypotenuse
f. angle of elevation

Take It to the NET
Online vocabulary quiz at **www.PHSchool.com**
Web Code: adj-1151

Skills and Concepts

11-1 Objectives

- To find square roots of numbers (p. 580)
- To classify real numbers (p. 581)

The square of an integer is a **perfect square.** The inverse of squaring a number is finding a **square root.** The symbol $\sqrt{\ }$ indicates the positive square root of a number. A number that cannot be expressed as the ratio of two integers $\frac{a}{b}$, where b is not zero, is **irrational.** If a positive integer is not a perfect square, its square root is irrational.

Simplify each square root.

7. $\sqrt{1}$ 8. $-\sqrt{16}$ 9. $\sqrt{49}$ 10. $\sqrt{64}$ 11. $-\sqrt{36}$

Estimate to the nearest integer.

12. $\sqrt{5}$ 13. $\sqrt{11}$ 14. $\sqrt{33}$ 15. $\sqrt{62}$ 16. $\sqrt{91}$

Identify each number as rational or irrational. Explain.

17. 0.55 18. $\sqrt{64}$ 19. $0.\overline{45}$ 20. $\sqrt{15}$ 21. 0.123123 . . .

22. Explain why 0.12122122212222 . . . is an irrational number.

11-2 Objectives

▼ To use the Pythagorean Theorem (p. 584)

▼ To identify right triangles (p. 586)

In a right triangle, the two shortest sides are the **legs.** The longest side, which is opposite the right angle, is the **hypotenuse.** The Pythagorean Theorem states that in any right triangle the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse ($a^2 + b^2 = c^2$).

Can you form a right triangle with the three lengths given? Show your work.

23. 1 mi, 3 mi, 3 mi

24. 8 yd, 15 yd, 17 yd

25. $\sqrt{6}$ ft, $\sqrt{10}$ ft, 4 ft

26. 30 m, 40 m, 50 m

11-3 Objectives

▼ To find the distance between two points using the Distance Formula (p. 592)

▼ To find the midpoint of a segment using the Midpoint Formula (p. 594)

The Distance Formula states that the **distance** d between any two points (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

The Midpoint Formula states that the **midpoint** of a line segment with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Find the distance between each pair of points. Round to the nearest tenth.

27. $(3, 0), (0, 2)$

28. $(-1, 7), (3, 10)$

29. $(4, -5), (-8, -1)$

30. $(-10, -12), (-8, -11)$

31. $(2, -14), (9, -20)$

32. $(10, 4), (-2, -2)$

Find the midpoint of each segment with the given endpoints.

33. $H(0, 1)$ and $J(4, 7)$

34. $K(2, 6)$ and $L(4, 2)$

35. $M(-7, 8)$ and $P(3, -4)$

36. $A(4, 9)$ and $B(5, 11)$

37. $X(-15, -12)$ and $Y(-9, -4)$

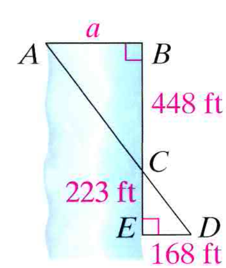
38. $D(20, 18)$ and $E(-15, -19)$

11-4 Objectives

▼ To write a proportion from similar triangles (p. 598)

You can write a proportion to solve indirect measurement problems using similar triangles.

39. Engineering An engineer needs to know what length to plan for a bridge across a river. She estimates the distance using the similar triangles $\triangle ABC$ and $\triangle DEC$ in the figure at the right. What is the distance a across the river?



11-5 Objectives

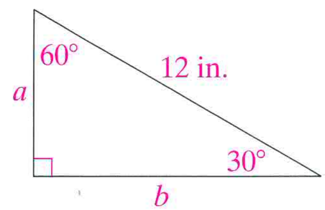
- ▼ To use the relationships in 45°-45°-90° triangles (p. 602)
- ▼ To use the relationships in 30°-60°-90° triangles (p. 603)

In a 45°-45°-90° triangle, the length of the hypotenuse is the length of a leg times $\sqrt{2}$.

In a 30°-60°-90° triangle, the length of the hypotenuse is 2 times the length of the shorter leg, and the length of the longer leg is the length of the shorter leg times $\sqrt{3}$.

Find the values of the variables, rounded to the nearest tenth, if necessary.

40.



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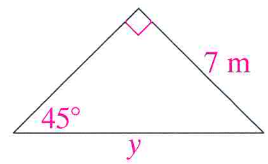

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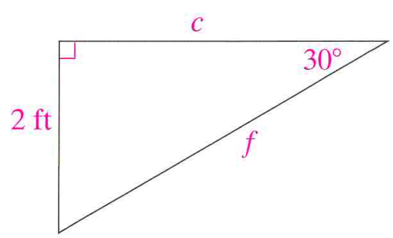

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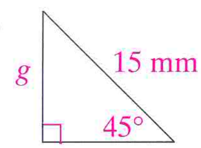


11-6 and 11-7 Objectives

- ▼ To find trigonometric ratios in right triangles (p. 608)
- ▼ To use trigonometric ratios to solve problems (p. 609)
- ▼ To use trigonometry to find angles of elevation (p. 614)
- ▼ To use trigonometry to find angles of depression (p. 616)

The ratios of the lengths of two sides of a right triangle are **trigonometric ratios.** Three trigonometric ratios are **sine, cosine,** and **tangent.** You can use these abbreviations when you find trigonometric ratios for a given acute $\angle N$.

$$\sin N = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos N = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan N = \frac{\text{opposite}}{\text{adjacent}}$$

An **angle of elevation** is formed by a horizontal line and a line of sight above it. An **angle of depression** is formed by a horizontal line and a line of sight below it.

Find each value. Round to four decimal places.

44. sin 16° **45.** tan 82° **46.** cos 25° **47.** tan 3° **48.** sin 87°

49. cos 73° **50.** cos 46° **51.** tan 45° **52.** sin 79° **53.** tan 13°

Solve each problem. Round to the nearest unit.

54. A loading ramp forms a 28° angle with the ground. If the base of the ramp is 15 ft long, how high does the ramp reach?

55. Melanie is flying a kite and lets out 100 ft of string. Rosa determines that from Melanie's hands the angle of elevation of the kite is 71°. Melanie's hands are 4.3 ft from the ground. What is the height of the kite?

Chapter 11

Chapter Test

Take It to the NET
Online chapter test at
www.PHSchool.com
Web Code: ada-1152

Simplify each square root.

1. $\sqrt{25}$ 2. $-\sqrt{81}$ 3. $\sqrt{100}$
4. $-\sqrt{4}$ 5. $\sqrt{16}$ 6. $\sqrt{49}$

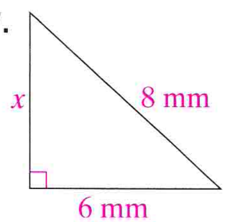
Estimate to the nearest integer.

7. $\sqrt{6}$ 8. $\sqrt{12}$ 9. $\sqrt{45}$
10. $\sqrt{78}$ 11. $\sqrt{85}$ 12. $\sqrt{118}$

Identify each number as rational or irrational.

13. 0.999 . . . 14. $\sqrt{24}$
15. $\sqrt{100}$ 16. 420,420

Find each missing length to the nearest tenth of a unit.

17.


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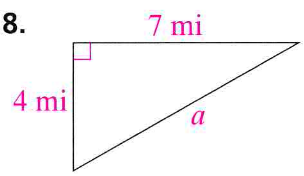

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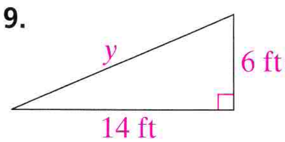

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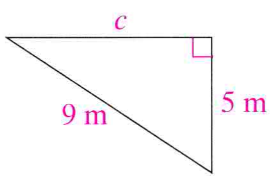


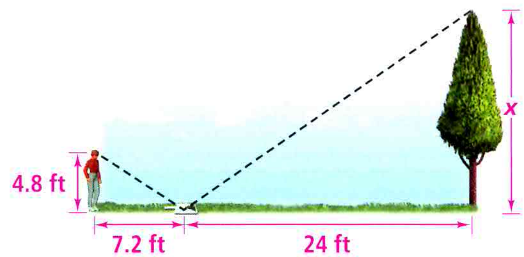
Find the distance between the points of each pair. Round to the nearest tenth.

21. $(0, 0), (4, 6)$ 22. $(5, -3), (-6, 2)$
23. $(-8, -9), (1, 2)$ 24. $(-1, -3), (-4, -7)$

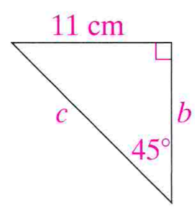
Find the midpoint of each segment with the given endpoints.

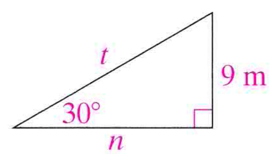
25. $C(5, 0)$ and $D(3, 6)$
26. $M(9, -4)$ and $P(2, 8)$

27. To estimate the height of a tree, Joan positions a mirror on the ground so she can see the top of the tree reflected in it. Joan's height, her distance from the mirror, and her line of sight to the mirror determine a triangle. The tree's height, its distance from the mirror, and the distance from the top of the tree to the mirror determine a similar triangle. Use the measurements below to find the height of the tree.



Find the missing lengths.

28.


29.


Find each value. Round to four decimal places.

30. sin 47° 31. tan 75° 32. cos 86°
33. tan 29° 34. cos 60° 35. sin 67°

36. **Writing in Math** Explain how a trigonometric ratio can be used to find a measurement indirectly.

37. **Navigation** The captain of a ship sights the top of a lighthouse at an angle of elevation of 12°. The captain knows that the top of the lighthouse is 24 m above sea level. What is the distance from the ship to the lighthouse?

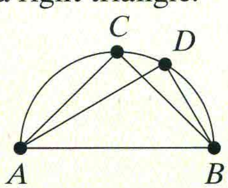
Test Prep

READING COMPREHENSION

Reading Comprehension **Read the passage below. Then answer the questions on the basis of what is *stated* or *implied* in the passage.**

Early Geometry The early Babylonians and Egyptians used practical geometry in their buildings, but it was a Greek named Thales who first wrote down the formal abstract geometry that we know today.

Thales, an olive-oil merchant, lived from about 600 to 550 B.C. One of the unchanging properties of triangles that he discovered was that a triangle drawn in a semicircle (half a circle), with the diameter as a hypotenuse, will always be a right triangle.



Around 540 B.C. a student of Thales, Pythagoras, founded a group that studied, among other things, mathematics. One of the rules of the *Pythagoreans* was never to eat beans! At first, they believed that the entire universe was made of only rational numbers, but working with right triangles convinced them that they could draw lines that have lengths equal to the square root of 2, the square root of 5, and so on.

1. What is the name of a teacher of Pythagoras?

2. What was the name of a group founded by Pythagoras?

3. What is the measure of $\angle C$?
A. 360° **B.** 180° **C.** 90° **D.** 45°

4. Which term best describes $\overline{AC}$?
F. leg **G.** sine
H. hypotenuse **I.** tangent

5. If $AC = CB = 1$ in., what is AB?
A. 2 in. **B.** $\sqrt{3}$ in. **C.** $\sqrt{2}$ in. **D.** 1 in.

6. If $AB = 6$ in. and $DB = 3$ in., what is AD?
F. $3\sqrt{2}$ in. **G.** $3\sqrt{3}$ in.
H. $6\sqrt{2}$ in. **I.** $6\sqrt{3}$ in.

7. If AB is $\sqrt{8}$ cm *and* $AC = CB$, what are AC and CB?

8. Which term does NOT apply to $\overline{AB}$?
A. diameter **B.** hypotenuse
C. leg **D.** line segment

9. How does the measure of $\angle D$ compare with the measure of $\angle C$?

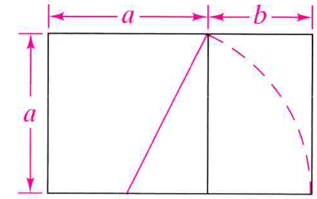
10. What country was Thales from?

11. How long did Thales live?

12. What must be true of $\sqrt{2}$?
I. It is not a rational number.
II. A segment can have length $\sqrt{2}$.
III. It is equal to $\sqrt{5}$.
F. I only **G.** I and II
H. I and III **I.** II and III only

Picture Perfect

Applying Ratios Ancient Greeks realized that rectangles with certain dimensions were especially pleasing to the eye. A golden rectangle has sides that form the proportion $\frac{a}{b} = \frac{a+b}{a}$. The ratio of two sides of a golden rectangle is called the *golden ratio.* Artists make paintings with dimensions close to the golden ratio. Photographers often *crop*, or cut, their photographs to be golden rectangles.

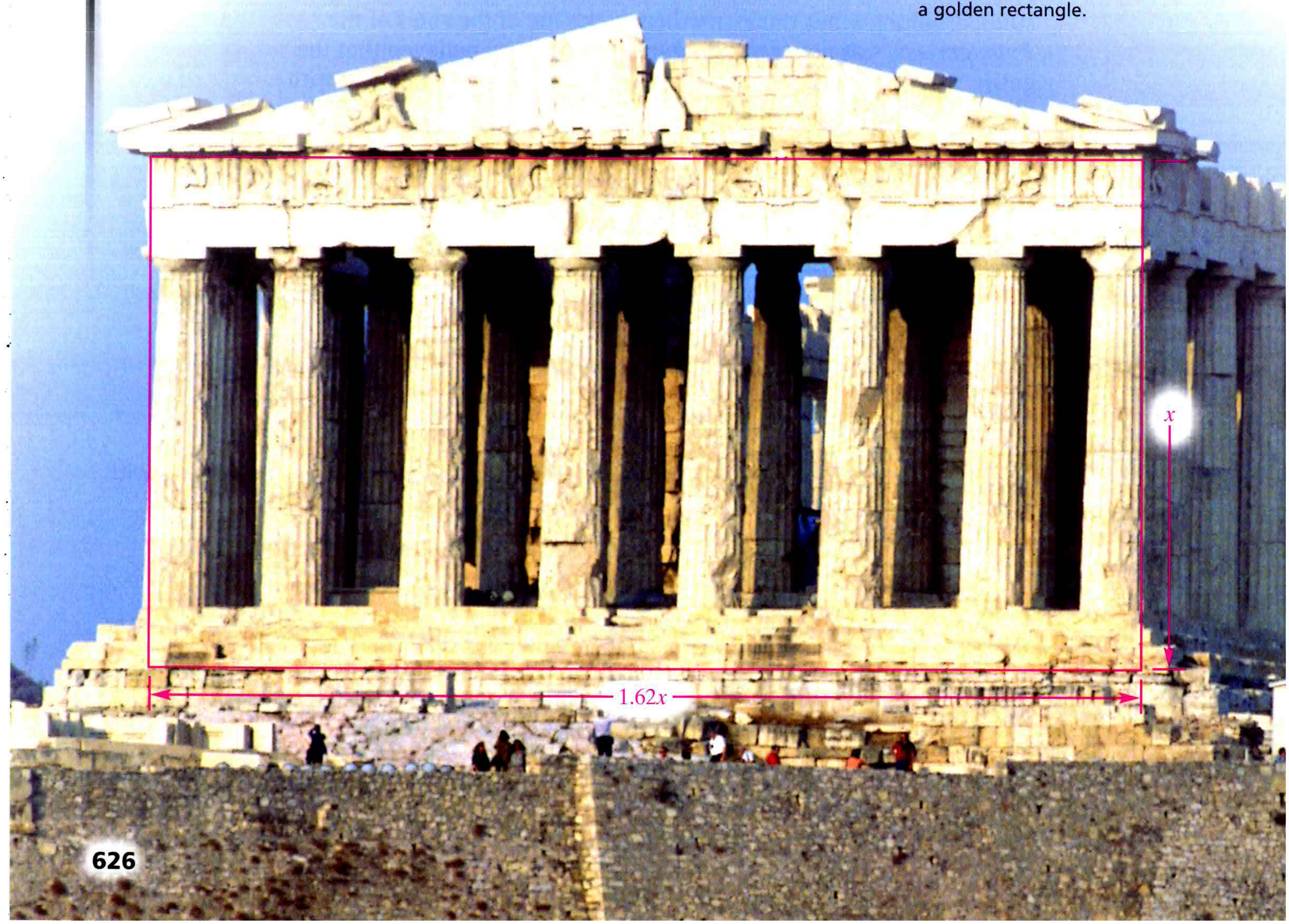


How to Make a Golden Rectangle

Start with a square with side length *a*. Open a compass the length from the midpoint of one side of the square to a corner on the opposite side. Make an arc. Extend the side containing the midpoint to intersect the arc. Make a rectangle using lengths *b* and *a*.

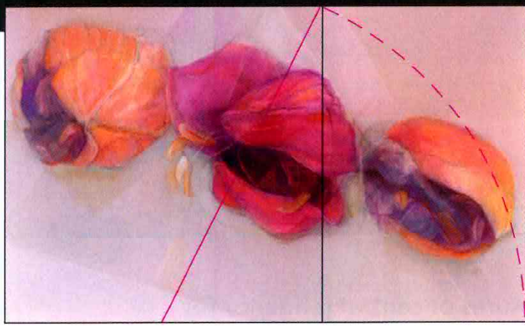
Golden Rectangle Proportion

a is to *b* as (*a* + *b*) is to *a*.



The Parthenon

The front of the Parthenon, in Athens, Greece, approximates a golden rectangle.



The Golden Rectangle in Art

Artist Sydney McGinley makes pastel paintings of flowers, such as *Pyramidal Fuschia*, using a golden rectangle to position the flowers within the painting.

The Villa Savoye

The Swiss architect Charles Le Corbusier designed the Villa Savoye, outside Paris, France, based on the golden rectangle, among other things. The house, built in 1930, looks different from each side.

Activity

1. **a.** Copy the table. For each rectangle, find the ratios $\frac{a}{b}$ and $\frac{a+b}{a}$. Write the ratios in decimal form, rounding to the nearest tenth. Write *yes* if the decimals are approximately equal.
   b. Which rectangles are close to a golden rectangle? Explain.

Rectangle Dimensions (inches)

Side a	Side $a + b$	$\frac{a}{b}$	$\frac{(a+b)}{a}$	Golden Ratio?
5	8	$\frac{5}{3} = 1.\overline{6} \approx 1.7$	$\frac{8}{5} = 1.6$	Yes
3	4	■	■	■
6	10	■	■	■
14	22	■	■	■
12	16	■	■	■
15	24	■	■	■
18	26	■	■	■
21	33	■	■	■

2. Photo labs make prints in several sizes, including 3×5, 4×6, and 8×10. Which of these sizes is closest to a golden rectangle?
3. **Open-Ended** From the table, choose one pair of dimensions that do not form a golden rectangle. Change the value of a to a whole number that makes the rectangle closer to a golden rectangle. Justify your choice by showing your work.
4. **Research** Find at least three images from magazines or catalogs that you think approximate golden rectangles. Measure the images and calculate the ratio of length to width.

Take It to the NET For more information about the golden ratio, go to **www.PHSchool.com**.
Web Code: ade-1153

Where You've Been

- In Chapter 3, you investigated measures of central tendency for sets of data.
- In Chapter 4, you simplified fractions by dividing the numerator and denominator by the GCF.
- In Chapter 5, you performed operations with fractions.
- In Chapter 6, you found the probability of events. You also wrote fractions and decimals as percents.

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Diagnosing Readiness (For help, go to the lesson in green.)

Finding the Median (Lesson 3-3)

Find the median.

1. 12, 14, 10, 9, 13, 12, 15, 12, 11

2. 55, 53, 67, 52, 50, 49, 51, 52, 52

3. 101, 100, 100, 105, 102, 101

4. 0.2, 0.5, 0.11, 0.25, 0.34, 0.19

Multiplying Fractions (Lesson 5-4)

Find each product.

5. $\frac{2}{3} \cdot \frac{1}{2}$ **6.** $\frac{7}{8} \cdot \frac{6}{7}$ **7.** $\frac{9}{10} \cdot \frac{8}{9}$ **8.** $\frac{5}{6} \cdot \frac{4}{5}$ **9.** $\frac{3}{4} \cdot \frac{2}{3}$ **10.** $\frac{7}{8} \cdot \frac{6}{7} \cdot \frac{5}{6}$

Finding Probability (Lesson 6-4)

Find the probability for one roll of a number cube.

11. $P(2)$ **12.** $P(5)$ **13.** $P(2 \text{ or } 5)$ **14.** $P(8)$

15. $P(1, 2, \text{or } 3)$ **16.** $P(\text{less than } 1)$ **17.** $P(\text{not } 3)$ **18.** $P(\text{greater than } 4)$

A student is chosen at random from a class of 15 boys and 18 girls. Find each probability.

19. $P(\text{girl})$ **20.** $P(\text{boy})$ **21.** $P(\text{not a girl})$ **22.** $P(\text{not a boy})$

Fractions, Decimals, and Percents (Lesson 6-5)

Write each percent as a decimal, and each decimal or fraction as a percent.

23. 50% **24.** 36% **25.** 20% **26.** 5%

27. $\frac{1}{5}$ **28.** $\frac{7}{8}$ **29.** 0.28 **30.** 0.3